THE EFFECT OF A DECLINE IN MORTALITY ON THE GROSS REPRODUCTION RATE

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During recent years the high mortality rates once existing in many Latin American countries have undergone extremely rapid declines. At the same time, moderate increases in fertility have occurred in most of these countries.[1] The question therefore arises as to whether the drastic fall in mortality, quite apart from the associated improvement in health, is tending to increase the fertility of Latin American populations.[2]

This paper will attempt to answer the question affirmatively by demonstrating that a drop in mortality automatically causes the gross reproduction rate[3] to rise. By definition, however, changes in mortality cannot directly affect the gross reproduction rate. The latter is based on the total female population in the reproductive ages and on the number of births within this population. It assumes that a woman lives through the reproductive period. Therefore, when mortality changes, but the age-specific fertility rates remain constant, the gross reproduction rate cannot vary.

Yet this theory does not consider the *indirect* effect of mortality decline on the mating system. If only those women living in unions[4] are having children,[5] the gross reproduction rate will vary with mortality even though the age-specific fertility within unions remains the same. A decline in mortality has the effect of expanding the life-span of unions and thus of increasing the proportion of women living in unions. The longevity of the union itself increases and,

consequently, so does the capacity to reproduce. A rise in fertility (measured in proportion to total female population) is possible even when the actual fertility *behavior* of reproductively active women does not change, or even slightly decreases.

The increase in fertility which can be ascribed to a lengthened timespan of unions is of relatively moderate magnitude. However, the size of many observed rises in the gross reproduction rate (and birth rate) which have occurred in conjunction with rapid mortality decreases are of this relatively moderate size, and thus could be subject almost entirely to the explanation discussed in this paper, and not to factors more directly influencing fertility.

The number of unions existing at the beginning of a given year, $t+1$, $({}_{t+1}U_{x+1})$, is:[6]

$$ {}_{t+1}U_{x+1} = {}_{t}U_{x}\ (p_{xy} - p_{\sigma}) + {}_{t}N_{x}\ p_{\sigma} $$

As can be seen, ${}_{t+1}U_{x+1}$ is determined by: 1. the cohort unions from the previous year *t*, $({}_{t}U_{x})$; 2. the total female population $({}_{t}N_{x})$; 3. the probability of joint survival (p_{xy}); and 4. the probability of a woman's entering a union during that year (p_{σ}). Now, if p_{σ} is assumed constant, the number of unions will be determined by p_{xy} (the effect of mortality) and by N_x (which will also depend upon mortality). Thus, assuming this to be true, in the above formula the variable which affects unions is mortality. As mortality decreases, ${}_{t}N_{x}$ increases, the probability of survival for both men and women increases and, therefore, so does the number of unions. Since the probability of survival for unions is the product of the probabilities of survival for men and women, a decrease in mortality has more effect on the probability of joint survival than on each of its components. Thus the relative increase of unions will be larger than the relative increase of either men or women. Therefore, the ratio of women living in unions to total female population will increase when mortality declines. Even when the variables affecting fertility (i.e., the mating age, the tendency to mate and the age-specific fertility rates of women in unions[7]) remain constant, the ratio of births to the total number of women of reproductive age will increase when mortality declines simply because the number of women in unions is increased proportionately.

A HYPOTHETICAL EXAMPLE

To measure how much it will change after a decline in mortality, the gross reproduction rate of two different populations will be calculated. The requirement of these two populations is that they differ only in previous mortality. In other words, both have the same base population any earlier year, but from that year until the present time these two populations have maintained constant, with the exception of mortality, all the variables affecting fertility. Thus, the demographic differences between the two populations are due exclusively to the decline of mortality. The proportion of women in unions to female population is different in the two populations. Therefore, by holding fertility rates within unions constant, two different gross reproduction rates will result.

For the purposes of illustration the population of Costa Rica in 1927 will be used, because of the high rates of both fertility and mortality prevailing there at that time. The constants all have to be estimated for this date, but this does not affect the result.[8] The estimates appear in the Appendix, along with the two projections of the female population, one with the previous assumed decline in mortality, the other with constant mortality.[9] Only projections of the female population and of the females in unions were made, since only these figures are essential for measuring effects on reproduction as discussed in this example.[10]

The estimation of the number of females in unions at the time when the two rates are going to be calculated was made by using the female population projections and the estimations of the age distribution of the females in unions at the beginning of the period. The following was done for each year:

$$ {}_tN_x - {}_tU_x = {}_tN_x $$

$$ {}_tU_x = {}_tN_x \cdot p_x $$

$$ {}_{t+1}U_{x+1} = {}_tU_x \cdot p_{xy} + {}_tU_x $$

Using two different sets of probabilities of joint survival (as per a declining or a constant mortality—see the Appendix), the female populations living in unions were obtained for the end of the ten-year period (see the Appendix and Table 6). As was expected, in

the population in which mortality declined, the proportion of unions was greater than in the population in which mortality was kept constant.[11] By multiplying the age-specific fertility rates of the female population in unions (see Table 5, column 3) by the number of unions, the number of births by age of mother (B_x) was determined:

$$B_x = U_x \cdot \emptyset_x$$

Then, dividing the number of births from mothers aged x by the total female population in the same age, the classical age-specific fertility rates ($\emptyset_x$) were obtained, as shown in Table 1.

$$\emptyset_x = \frac{B_x}{N_x}$$

After this, the gross reproduction rate was established, k having a value of .4878:

$$GRR = k \Sigma \emptyset_x$$

The results obtained for this example, as seen in Table 1, show an increase of age-specific fertility in all age groups, and thus an increase in the gross reproduction rate.

A Test on Several Countries

The magnitude of the variation in the gross reproduction rate when mortality decreases will depend upon the original mortality level and on the degree of the decline. If a sudden decline in mortality is experienced in a population in which fertility and mortality are high, a relatively large increase in the gross reproduction rate is to be expected. If mortality is already at a low level before the decline, the observable effect will be negligible. If the decline is slow (even from a high level) the same effect will occur (if other fertility variables are constant over time), but the short-term manifestations will be insignificant.

In Latin American countries mortality declined very rapidly during 1950–60.[12] For a few of these countries where census data are available,[13] the child-woman ratios and the proportion of women living in unions to the total female population age 15 and over, or 15 to 49, are given in Table 2. In every country tested the proportion of females living in unions and the child-woman ratios increased. The mortality declines can be considered one of the main

TABLE I. AGE-SPECIFIC FERTILITY RATES,* AND GROSS REPRODUCTION RATES

Age Groups	*Population with Hypothesis of Constant Mortality*	*Population with Hypothesis of Declining Mortality*	*Per cent Change*
15–19	.565	.571	1.0
20–24	1.420	1.451	2.2
25–29	1.665	1.721	3.4
30–34	1.458	1.529	4.8
35–39	.989	1.053	6.5
40–44	.469	.509	8.3
45–49	.100	.109	9.1
Gross reproduction rate	3.25	3.39	4.1

* $\phi_{x,x+5} = \sum_{n=0}^{5} \phi_{x+n}$

TABLE 2. CHILD-WOMAN RATIOS AND PROPORTION OF FEMALE POPULATION LIVING IN UNIONS IN SELECTED COUNTRIES

		Child-Woman Ratios		*Proportion of Females Living in Unions*	
Country	*Year*	*Children Age 0–4 per Woman 15–44*	*Children Age 5–9 per Woman 20–49*	*Age 15 and Over*	*Age 15 to 49*
Dominican Republic	1950	.798	.789	.580	
	1960	.900	.949	.609	
El Salvador	1950	.674	.691	.510	.532
	1961	.803	.836	.528	.555
Ecuador	1950	.765	.762	.556	.580
	1962	.815	.880	.598	.617
Honduras	1950	.727	.698	.461	
	1961	.953	.925	.570	
Mexico 14+	1950	.684	.740	.571	
	1960	.787	.862	.574	
Nicaragua 14+	1950	.702	.767	.482	
	1963	.890	1.014	.565	
Panama	1940	.641	.688	.505	
	1950	.752	.765	.566	.598
	1960	.813	.823	.570	.597
Venezuela	1950	.767	.731	.485	.531
	1961	.880	.866	.537	.567
Theoretical example constant mortality					.517
declining mortality					.542

causes for the increase in the ratio of unions to population and the rise in fertility.

One of these countries, Venezuela, has enough census data[14] for 1950 and 1961 to permit a more detailed analysis of the rise in the child-woman ratio connected with a fall in mortality. For this country the cumulative fertility rates[14] by age groups for all women, as well as only for mothers,[15] were calculated. The fertility of the latter group, in fact, decreased during the intercensal period, which may indicate a slight change in reproductive behavior within unions, tending toward smaller numbers of children per couple. On the other hand, the fertility of *all* women increased during the intercensal period. This occurred because the proportion of women aged 12 years and over having children (living in unions) was higher in 1961 than in 1950. The same phenomenon can be found in almost every age group (Table 3).[16]

In general, the effect of a decline in mortality on the age-specific fertility rates of the total female population will vary with age. The higher the age, the bigger the increase in the age-specific fertility rates of the total female population. This is due to the increase of the joint survival ratios—and hence the number of unions saved

TABLE 3. CUMULATIVE FERTILITY RATES BY AGE, VENEZUELA

	1961			*1950*		
Age	*I**	*II†*	*III***	*I**	*II†*	*III***
12+	4.65	2.81	60.4	4.74	2.69	56.5
12–14	1.04		.3	1.12		.2
15–19	1.51	.29	19.2	1.44	.25	17.6
20–24	2.50	1.53	61.5	2.37	1.35	56.7
25–29	3.71	2.83	76.9	3.58	2.59	72.4
30–34	4.66	3.65	84.3	4.70	3.61	76.8
35–39	5.38	4.59	85.6	5.59	44.1	78.8
40–44	5.93	4.92	83.1	6.05	4.71	77.8
45–49	5.96	5.00	83.9	6.33	4.93	78.0
50+	5.62	4.61	82.3	6.34	4.81	75.8

* Average of children ever born to mothers.
† Average of children per female population.
** Percent of female population reported as mothers.

from dissolution due to death of one or both partners—which will be bigger in the latter than in the earlier part of the fertile period.[17] If age-specific fertility rates of women living in unions does not change, fertility of total female population will change, mainly as an increase of late fertility. This can be seen in the hypothetical example presented here, summarized in Table 1.

CONCLUSION

The gross reproduction rate has been shown, both theoretically and actually, to be indirectly affected by changes in mortality, because the joint survival of couples is increased and, therefore, the proportion of women living in unions to total female popuation is increased. With other factors affecting fertility held constant, this in turn will increase the proportion of births to total female population. The result will be a rise in several measures of fertility: in the gross reproduction rate, in child-woman ratio and usually in crude birth rate.[18]

If the gross reproduction rate decreases (or fails to increase), it will do so because some factors bringing about a decline in fertility are countering the effect of declining mortality discussed here. Conversely, if a very large increase in the gross reproduction rate occurs, other factors are acting jointly with the mortality effect to bring about a large part of the increase, since the increase to be expected from the mortality effect alone is relatively modest.

Finally, in explaining rising fertility rates (principally in the latter part of the fertile period) for a population in which mortality is changing downward from high levels, the effect of a decline in mortality on the survival of unions—with the concomitant effect on fertility—should first be established, and then other causes should be examined.

Declining mortality seems to be one of the causes of the recent fertility increases in Latin American countries, and, hopefully, these countries will start to decline in fertility, as has been observed in other cases once mortality decreases.[19]

APPENDIX

An explanation of the estimation of marital status distribution, the age-specific fertility rates within unions, the probabilities of entering into unions, ages of men and women in unions and probabilities of joint survival for the 1927 Costa Rica population are given below. Calculations for female populations and women living in unions after a ten-year period of different mortality patterns are also given.

Marital Status Distribution

A comparison of the "marital status" distribution of 1927, with the same distribution in the 1950 and 1963 censuses (which specifically included consensual unions), showed that those in consensual unions in 1927 were in fact tabulated as single persons (Table 4). With this established, a comparison of the three censuses allowed an estimation of the total number of consensual unions in 1927. Similarly, the proportion of women living in unions to total women in each age group in 1950 and 1963 provided a basis for an estimation of the same proportion for 1927. The distribution by age of this proportion was smoothed graphically, and the values for single years were taken from the graph (Figure 1 and Table 5).

Age-Specific Fertility Rates of Unions ($\emptyset_x$)

Age-specific fertility rates of unions were estimated from census and vital statistics data for 1963.[20] The census provides the number of unions and the birth register reveals the number of births by age of the mother. An average was taken of three years of birth data and this figure was applied to the number of unions by age of the women, allowing an estimation of $\emptyset_x$ by five-year age groups. By drawing a graph of these values, single-year values were determined (Figure 2 and Table 5, column 3).

Probability of Entering into Unions (p_x)

The estimation of p_x was also made using vital statistics and census information for 1963. The denominator of this probability, the number of women not in unions (single, widowed, divorced, separated), was taken from the 1963

TABLE 4. COSTA RICA—DISTRIBUTION OF FEMALE POPULATION 15 YEARS OLD AND OVER BY MARITAL STATUS (PERCENT)

	YEARS			
Marital Status	*1927*	*1950*	*1963*	*Estimated 1927*
Single	45.3	38.6	35.1	37.8
Married	42.7	43.1	47.2	42.7
Consensual union		7.4	7.6	7.5
Separate	1.8	2.4	2.8	1.8
Widowed	9.9	8.0	6.5	9.9
Divorced	.3	.4	.7	.3
Total	100.0	100.0	100.0	100.0

FIGURE I. PERCENTAGE OF FEMALE POPULATION LIVING IN UNIONS, BY AGE.

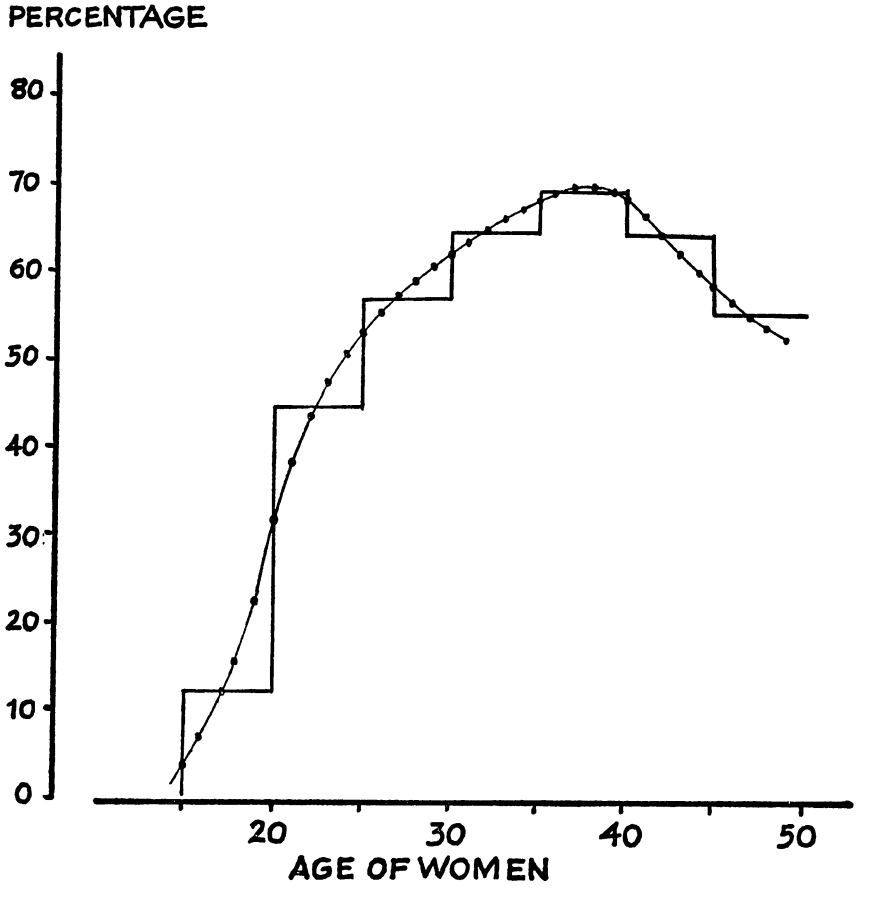
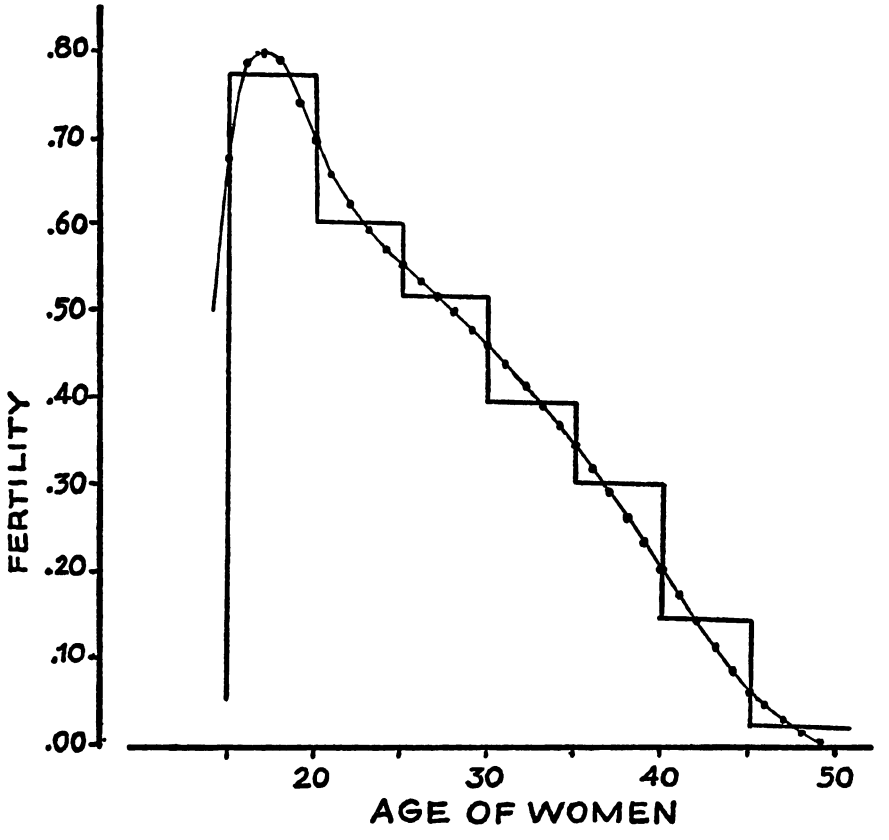


TABLE 5. ESTIMATES OF SELECTED DEMOGRAPHIC CHARACTERISTICS, COSTA RICA, 1927

Age of Women (x)	*Women Living in Unions* (U_x)	*Fertility Rates of Women in Unions* (Q_x)	*Female Probability of Entering into Union* (p_x)	*Mean Age of Men in Unions* (y)
15	234	.673	.0228	23
16	394	.787	.0432	24
17	638	.803	.0648	24
18	790	.785	.0850	24
19	1,128	.742	.1026	25
20	1,531	.689	.1122	26
21	1,746	.650	.1197	27
22	1,924	.615	.1221	27
23	2,041	.587	.1208	28
24	2,094	.569	.1153	29
25	2,138	.551	.1088	30
26	2,156	.533	.1011	31
27	2,151	.516	.0932	32
28	2,095	.497	.0852	33
29	2,086	.479	.0776	34
30	2,039	.460	.0700	35
31	1,991	.439	.0628	36
32	1,945	.416	.0566	37
33	1,907	.392	.0516	38
34	1,870	.367	.0470	39
35	1,836	.344	.0430	40
36	1,798	.318	.0396	41
37	1,756	.292	.0364	42
38	1,711	.263	.0335	43
39	1,645	.232	.0308	44
40	1,577	.201	.0283	45
41	1,483	.170	.0260	46
42	1,391	.142	.0238	47
43	1,302	.113	.0217	47
44	1,205	.088	.0197	48
45	1,120	.064	.0178	49
46	1,044	.046	.0160	50
47	975	.030	.0144	51
48	905	.015	.0130	52
49	828	.003	.0117	53

FIGURE 2. AGE-SPECIFIC FERTILITY RATES OF WOMEN IN UNIONS ($\emptyset_w$).



census. For the numerator, the number of women entering unions during 1963, an estimation was necessary. The vital statistics give the number of marriages by age of women and men, but no information is available for consensual unions. The age distribution of women entering into consensual unions in 1963 was assumed to be the same as for those who married, and the ratio of consensual unions to marriages in 1963 was also assumed to be proportionally the same as that given in the census.[21] That is:

$$z = \frac{M + CU}{M}$$

then

$$U^{1963} = z \cdot M^{1963}$$

M is the total married female population and CU is the total female population in consensual unions in the census.

Then, taking the number of women who married in 1963 (M^{1963}) from the vital statistics register,[22] and multiplying by z, the total number of unions entered in 1963 (U^{1963}) was estimated. These estimated unions were distributed by age in the same proportions as legal marriages in 1963. Taking both the population entering into unions and the population not living in unions by five-year age groups, the following probabilities were calculated:

$$p_{x,x+5} = \frac{U_{x,x+5}}{N_{x,x+5}}$$

From this a graph was drawn and the single-year probabilities taken therefrom (Figure 3 and Table 5, column 4).

Age of Women and Men in Each Union (x and y)

In this case, the problem was to find the average age of men (y), given age x of their mates. This information is not available in the Costa Rica census, but the tabulation of marriages during each year by age of males and females is available in the vital statistics yearbook. Therefore, assuming that the age in consensual unions is the same as in marriages, and that the difference in age between the partners has not changed over the years, the following procedure is acceptable: For the mean age of each five-year age group of females a corresponding mean age for their mates can be found. The differences between the mean ages of men and women will apply only to those entering unions during the base year, not to the whole population. But assuming the differences of ages and the per cent distribution of unions by female age to be constant through time, a Lexis diagram allows an estimation of the age difference within all unions.[23] The mean age difference between men and women in all unions was then plotted on a graph (Figure 4). To avoid lengthy probability calculations, these corresponding male single ages were rounded to the nearest integer year (Table 5, column 5).

Probability of Joint Survival by Single Years (p_{xy})

Having once established the average age of both partners in the unions, the probability of survival for female and male, p_x and p_y, must next be estimated to obtain p_{xy}. No life table was available for the Costa Rican population of 1927, but life expectancy at birth was approximately 40 years at that time. The United Nations Life Tables level 40[24] was considered an adequate substitute for the 1927 Costa Rica life table. This was established after a comparison between the available Costa Rica Life Table, 1950,[25] and the United Nations Life Table Models showed virtually the same mortality pattern. Therefore, United Nations Model Tables were used for the estimation of p_x and p_y. Since the period under study was ten years in length, L_x values[26] (by single ages) from ten life tables were required.

In selecting proper model life tables, particular attention was paid to the decrease in mortality. The life expectancy during the ten-year period was assumed to have increased from 40 to 51.25 years. To obtain the ten life tables, the model life tables were linearly interpolated.[27] Then, using Beers' coefficients,[28] the initial values of ${}_5L_x$ were broken down into single ages (L_x). The L_x values found allowed the calculation of p_x and p_y and, therefore, p_{xy} by considering the mean ages of men and women living in unions (from Table 5, columns 1 and 5).[29]

Projection of Female Population and Women Living in Unions

The female population by single age at the beginning of the ten-year period (Table 6, Column 2) was projected under two different mortality hypotheses: one constant[30] and the other of declining mortality.[25] To establish the number of women living in unions at the end of the ten-year period the procedure explained in the text was followed, by using the distribution of women in unions at the beginning of the period, as shown in Table 5, column 2, the female population in the ten years of the projection, the joint survival ratios (p_{xy}) and the probability of women entering into unions (p_x, Table 5, column 4). The two different female populations and women living in unions, after the different mortality patterns during the same period of time, are shown in Table 6.

FIGURE 3. PROBABILITY OF A WOMAN AT AGE X ENTERING THE UNION DURING A YEAR (p_x).

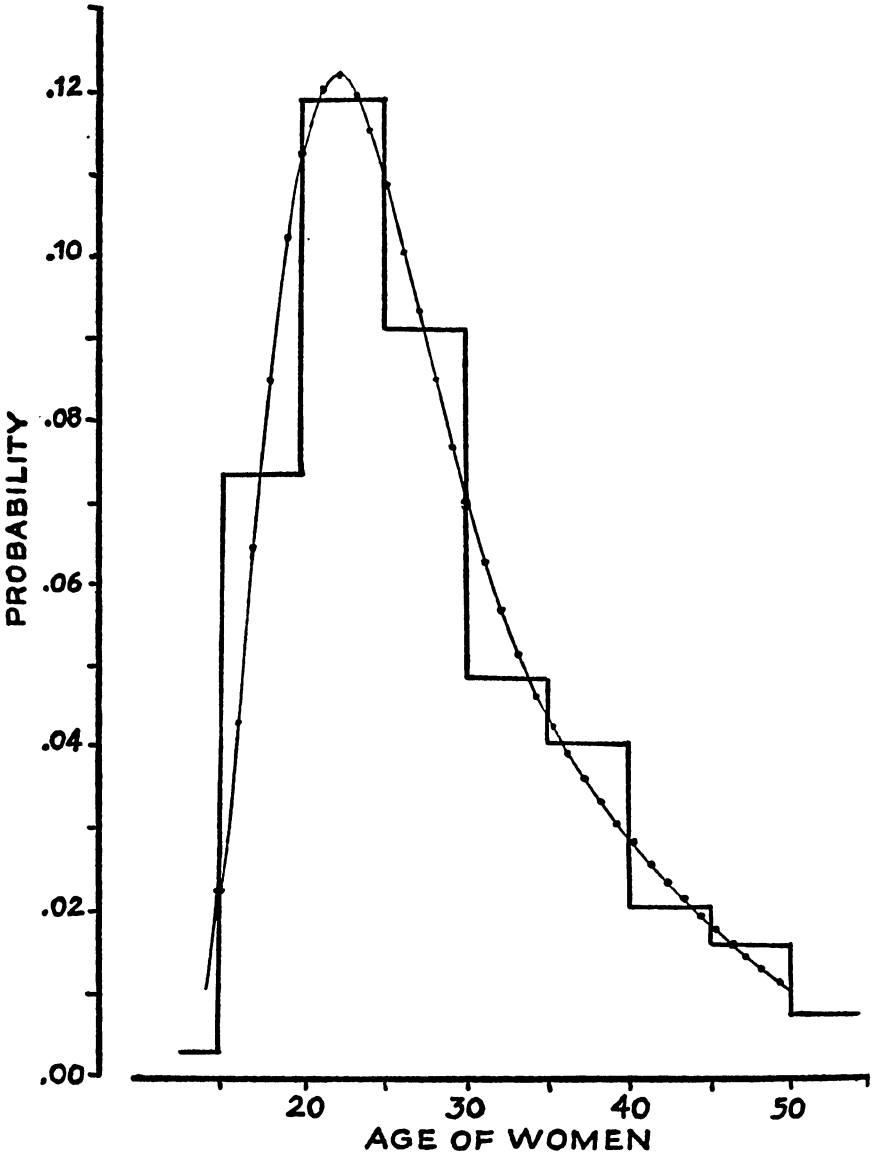


FIGURE 4. DIFFERENCES OF MEAN AGES OF MEN AND WOMEN IN UNIONS.

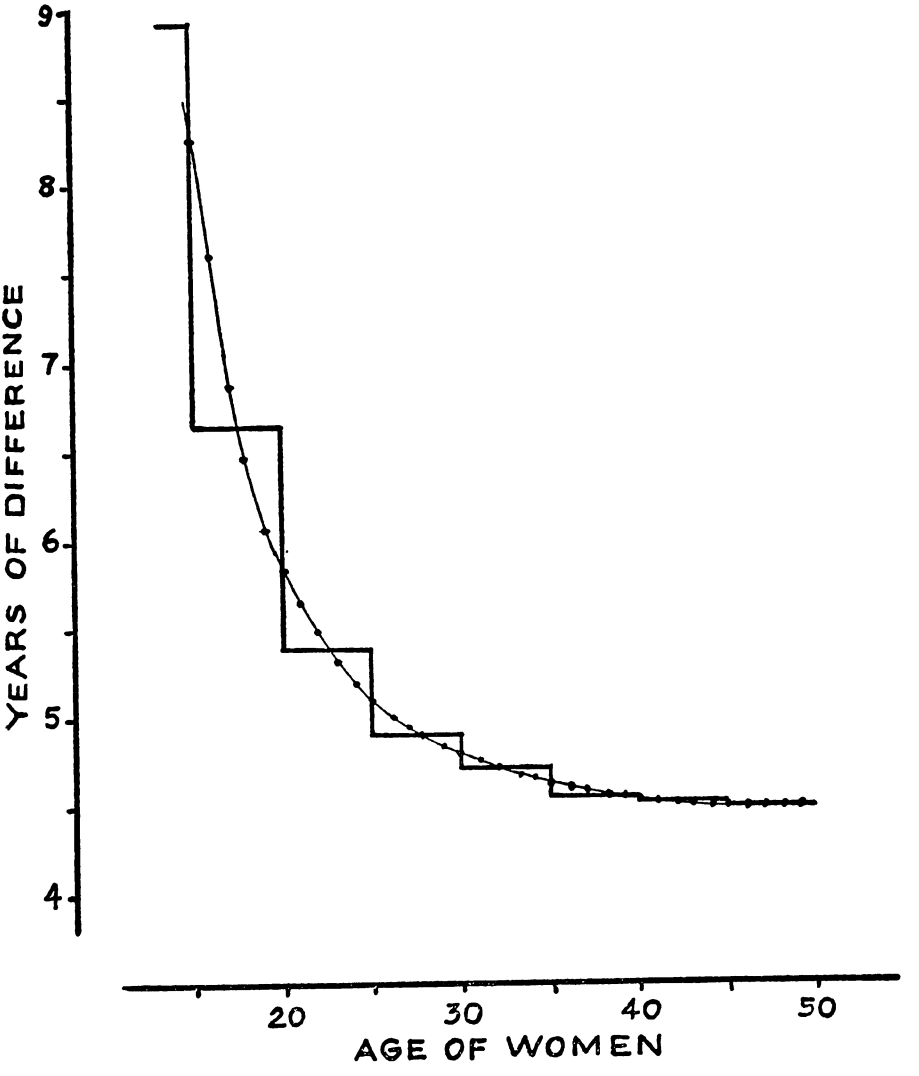


TABLE 6. PROJECTIONS OF FEMALE POPULATIONS AND WOMEN LIVING IN UNIONS

Age	*Female Population at the Beginning of the Period*	*Findings After a Ten-Year Period of Constant Mortality*		*Findings After a Ten-Year Period of Declining Mortality*	
		Female Population	*Women Living in Unions*	*Female Population*	*Women Living in Unions*
5	6,613				
6	6,340				
7	6,152				
8	6,054				
9	6,034				
10	6,061				
11	6,087				
12	6,060				
13	5,943				
14	5,761				
15	5,570	6,163	259	6,219	262
16	5,400	5,966	481	6,028	488
17	5,229	5,824	791	5,897	806
18	5,064	5,740	1,165	5,792	1,192
19	4,904	5,707	1,580	5,788	1,623
20	4,739	5,702	1,984	5,789	2,049
21	4,571	5,692	2,354	5,785	2,437
22	4,413	5,634	2,647	5,736	2,748
23	4,270	5,500	2,850	5,562	2,966
24	4,138	5,309	2,975	5,417	3,126
25	4,012	5,111	3,038	5,220	3,200
26	3,884	4,933	3,135	5,044	3,306
27	3,748	4,757	3,169	4,869	3,353
28	3,599	4,592	3,069	4,703	3,254
29	3,442	4,435	2,990	4,543	3,179
30	3,284	4,275	2,936	4,379	3,139
31	3,135	4,114	2,897	4,220	3,103
32	3,002	3,963	2,810	4,068	3,029
33	2,889	3,827	2,729	3,931	2,946
34	2,791	3,702	2,609	3,805	2,831
35	2,700	3,584	2,506	3,685	2,727
36	2,610	3,465	2,390	3,562	2,610
37	2,527	3,338	2,293	3,434	2,510
38	2,451	3,200	2,136	3,367	2,350
39	2,380	3,056	2,019	3,145	2,237
40	2,313	2,910	1,930	3,000	2,149
41	2,247	2,773	1,830	2,858	2,041
42	2,177	2,650	1,741	2,732	1,944
43	2,100	2,545	1,665	2,625	1,579
44	2,018	2,453	1,593	2,556	1,807
45	1,938	2,367	1,531	2,444	1,719
46	1,858	2,281	1,434	2,538	1,630
47	1,773	2,201	1,371	2,266	1,533
48	1,682	2,127	1,296	2,244	1,467
49	1,587	2,056	1,223	2,105	1,335

REFERENCES

[1] This phenomenon has been observed, for example, in the Dominican Republic, El Salvador, Ecuador, Honduras, Mexico, Nicaragua, Panama and Venezuela.

[2] Ridley, Sheps, Lingner and Menken have shown magnificently, by using a simulation model, that when mortality declines, natality tends to increase. *See* Ridley, Jeanne C., Sheps, Mindel C., Lingner, Joan W. and Menkin, Jane A., The Effects of Changing Mortality on Natality, *Milbank Memorial Fund Quarterly,* 45, 77–93, January 1967.

[3] The gross reproduction rate is defined as:

$$GRR = k \sum_{15}^{49} \emptyset_x$$

where $\emptyset_x$ are the age-specific fertility rates for the total female population, and k is the proportion of females' births to total births.

[4] "Union" is defined as any sexual relationship between a man and a woman that will be terminated only by the death of one or both partners.

[5] The assumption that all children born are products of unions, and conversely that women outside of unions are not contributing to reproduction, will not have an effect on the validity of the conclusions, since the actual number of births outside unions is not significant.

[6] Symbols used in this paper, and the deduction of the formula are:

- ${}_tN_x$ Number of women x to x + 1 years of age at the beginning of year t
- ${}_tU_x$ Number of women x to x + 1 years of age in unions at the beginning of year t
- ${}_tU_x$ Number of women x to x + 1 years of age entering unions during the year t
- ${}_tN_x$ Number of women x to x + 1 years of age not in unions at the beginning of year t
- $\emptyset_x$ Specific fertility rate of female population at age x to x + 1
- $\emptyset_a$ Specific fertility rate of females aged x to x + 1 in unions
- p_a Probability of a woman at age x to x + 1 entering the union during a year

$$p_a = \frac{{}_tU_a}{{}_tN_a}$$

- p_x Probability that a woman at age x to x + 1 will live one year longer
- p_y Probability that a man at age y to y + 1 will live one year longer
- p_{xy} Probability that a woman at age x to x + 1 and a man at age y to y + 1 will live one year longer

$$p_{xy} = p_x \cdot p_y$$

With these definitions established the effect of mortality on length of unions can be formulated. The number of unions at the beginning of the year t + 1 will be:

$${}_{t+1}U_{x+1} = {}_tU_x \cdot p_{xy} + {}_tU_a$$

But,

$${}_tU_a = {}_tN_a p_a$$

and

$${}_tN_a = {}_tN_x - {}_tU_x$$

Therefore

$${}_{t+1}U_{x+1} = {}_tU_x\ (p_{xy} - p_a) + {}_tN_x p_a$$

[7] Only unions will contribute to the gross reproduction rate since it has already been stipulated that women outside of unions are not reproducing. When the death of either partner occurs, the union is dissolved, and its contribution to the gross reproduction rate becomes zero.

[8] In Costa Rica, as in all Latin American countries, consensual unions exist as well as legal marriages, and carry out the same functions regarding reproduction. These consensual unions will be considered, along with legal marriages, as being within the definition of unions. Unfortunately, the 1927 Costa Rica Census did not include a tabulation of consensual unions. In addition to the need for estimating this figure, lack and/or scarcity of data makes necessary the estimate of U_x, p_{x}, p_{xy}, $\emptyset_x$ and $\emptyset_{x}$. At any rate, this will not be "the case of Costa Rica." It will simply be a hypothetical case beginning with conditions very similar to those in Costa Rica in 1927, but with subsequent variations for purposes of analysis.

[9] The decline in mortality assumed increased expectation of life at birth from 40.00 years to 51.25 years.

[10] Because the projections were for ten years only, the entire population was not included, but only that group from five to forty-nine years of age. In this way the fertile female population was determined over a ten-year period under the two mortality assumptions. Before attempting these projections, the ten-year age group in he 1927 census was divided into quinquenial groups (except the five to nine group) using the following formulae:

$$Q_{x,\ x+5} = \frac{1}{24}(-S_{x-15,\ x-5} + 11S_{x-5,\ x+5} + 2S_{x+5,\ x+15})$$

when the decennial group that is being separated is central; and

$$Q_{x,\ x+5} = \frac{1}{24}(8S_{x-15,\ x-5} + 5S_{x-5,\ x+5} - S_{x+5,\ x+15})$$

when the decennial age group corresponds to an extreme group (the youngest group). Single ages were then obtained using Beers' Coefficients.

[11] The proportion of females living in unions to the total female population age 15 to 45 years at the end of the period of projection were .517 and .542 for the projections of constant and declining mortality, respectively.

[12] The gain of life expectancy at birth in this period has been ten years in almost every case.

[13] It was considered preferable to use census information, because vital statistics from these countries in 1950 were not very complete.

[14] Ministerio de Fomento, Direccion General de Estadistica y Censos Nacionales, Octavo Censo General de Población 1956, Caracas, Venezuela, 1957, Table 124, p. 607; and Censo de Viviendas y Población 1961, Resultados Nacionales y del Area Metropolitana de Caracas (pre-tabulaciones por muestreo), Caracas, 1963, p. 1018.

[15] This is the closest classification available in the Venezuelan Census to the definition of women in unions. In effect, it is very close to this definition.

[16] In ages under 30, the small increase in the proportion of females living in unions could also be due to a slight change toward an early age at marriage, as has been observed during 1950–1960.

[17] At least when the decline of mortality is moving in a level of expectation of life under 65 years.

[18] Higher fertility resulting from improved health conditions associated with mortality decline (thus fewer stillbirths and other possible effects on fertility) has not been considered. Only the effect on fertility of lengthened life-span of unions is being established.

[19] *See* Davis, Kingsley, The World Demographic Transition, *The Annals of the American Academy of Political and Social Science,* 237, 1–11, January, 1945; and ———, The Theory of Change and Response in Modern Demographic History, *Population Index,* 29, 345–366, October, 1963.

[20] Direccion General de Estadistica y Censos, Ministerio de Economia y Hacienda, CENSOS DE 1963 POBLACION Y VIVIENDA, RESULTADOS PROVISIONALES OBTENIDOS POR MUESTRO, San Jose, Costa Rica, 1964; and ———, PRINCIPALES HECHOS VITALES OCURRIDOS EN COSTA RICA, San Jose, 1962, 1963, 1964.

[21] Of course, the age of the population entering into consensual unions is, on the average, somewhat younger than that of those entering into legal marriages. But because the proportion of unions which consists of consensual unions is very low (around 7.5 per cent, as shown in Table 4), and since perfect accuracy in this regard will not have an appreciable effect on the subject of study, the assumption is acceptable.

[22] Direccion General de Estadistica y Censos, 1963, *op. cit.*

[23] For Lexis diagram *see* Pressat, Roland, L'ANALYSE DEMOGRAPHIQUE, Paris, Presses Universitaires de France, 1961, pp. 16–58.

[24] United Nations, *Methods for Population Projections by Sex and Age,* Manual III St/SOA/Series A, Population Studies, No. 25, New York, 1956, pp. 78–79.

[25] Direccion General de Estadistica y Censos, Ministerio de Economia y Hacienda, *Tablas de Vida Costa Rica 1949–1951,* San Jose, 1957.

[26] Because the population from census is more comparable with the L_x than with the l_x, the probabilities were calculated as:

$$p_x = \frac{L_{x+1}}{L_x}$$

[27] The ten levels of expectation of life for both sexes were 40.00, 41.25, 42.50, 43.75, 45.00, 46.25, 47.50, 48.75, 50.00 and 51.25.

[28] Beers, Henry S., Six-Term Formulas for Routine Actuarial Interpolation, *Record of the American Institute of Actuaries,* 34, 60, June, 1945.

[29] This is only an example of method, not a precise attempt to analyze Costa Rican data. The graphical estimations herein are admittedly rough. However, the results obtained can be compared with actual values for Costa Rica in 1927 as follows: By applying the single-age-specific rates of unions in 1963 (assumed the same for 1927) to the distribution of women in unions estimated by age for 1927, the total births and hence the birth rate for 1927 were established. Now, as is well known, the official birth figures for Costa Rica in 1927 contain omissions. Consequently, the figure obtained by the above procedure was compared with an estimation already made which corrected the omission and gave the following birth rates per thousand: 1920–24, 44.9; 1925–29, 44.6; (Collver, Andrew, BIRTH RATES IN LATIN AMERICA, Berkeley, Institute of International Studies, University of California, 1965). The birth rate herein obtained was

45.5. The difference between the two is insignificant. Therefore, the values obtained here are acceptable as belonging to a population such as Costa Rica had in 1927.

[30] The female survival ratios (p_x) applied during the ten years were those pertaining to a life expectancy of 40 years for both sexes.

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