THE ELEMENTS OF STOCHASTIC PROCESSES WITH APPLICATION TO THE NATURAL SCIENCES

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A *stochastic process* is defined, in general, as an observable process developing in time at least partially under the influence of random or "chance" events. Conversely, a process that is known to operate in regular and nondeviating fashion according to some fixed rule is *deterministic*. Clearly, the waxing and waning of a gambler's holdings describe a stochastic process, whereas the growth of a sum of money deposited in a saving accounts is deterministic. In the latter case, the value of the principal is increased regularly by a fixed percentage (interest rate) per unit of time, while in the former the holdings are subject to the laws of chance. Similarly, but perhaps less obviously, the transmission of a mutant gene through the generations, or analogously, the passage of a rumor or piece of information from one individual to another in a population is stochastic, or probabilistic, in nature. Most systems involving man and his environment are, in fact, readily characterizable as stochastic processes. It is with the mathematics of such systems that N. T. J. Bailey's book is concerned.

Among the several publications that have been issued in recent years on the theory of stochastic processes, Bailey's book is unsurpassed in its ability to communicate the impressive scope and utility of the stochastic method to the mathematically unsophisticated. His exposition of the major techniques and results of the theory of elementary stochastic processes is mathematically sound, and his choice of selected topics and applications in the physical sciences is especially useful to those wishing to gain insight into possible applications in their own areas. This book is by far the best currently available at the elementary level, and the value of the techniques discussed therein is by no means restricted to those engaged in work in the physical sciences.

The concepts involved in the application of stochastic processes are relatively simple but these simple ideas often prove difficult to manipulate mathematically. Bailey states with commendable honesty that if his book is to be used as a text for a course in elementary stochastic processes an acquaintance with the elements of probability theory and basic statistics is necessary, along with some knowledge of matrix algebra, complex variable theory, and ordinary and partial differential equations. While this is a sobering truth for anyone interested in learning the theory involved, it should not dismay the person whose main interest is in learning something of the applications, since the interesting results for most models are clearly stated and indexed and references are given to applications in the existing literature. The author himself best characterizes the stochastic approach by stating that "the whole philosophy of the present volume is essentially rooted in real phenomena: in the attempt to deepen insight into the real world by developing adequate mathematical description of processes involving a flow of events in time, and exhibiting such characteristics as birth, death, growth, transformation, etc."

Following a section in which he reviews the mathematical techniques necessary for the development of the theory, Bailey goes on to describe some of the more interesting stochastic processes and their applications. One such, for example, is the theory of *recurrent events*. By considering a repetitive series of experimental trials, such as the consecutive flips of a coin, and defining an *event* as a particular combination of outcomes of these trials, e.g., the obtaining of exactly two heads in a row, it is possible to investigate the probability of occurrence of this event at least once in a given number of trials and to say something about the distribution of *waiting* or *recurrence* times between consecutive events of this nature. An interesting application of his theory to demography, for example, is one in which the event is defined as the occurrence of a live birth to a given human female. The waiting or recurrence time is then equivalent to the interval between successive live births for the female, a variable of some practical interest, and the distribution of these birth intervals can then be studied thoroughly, using the theory of recurrent events as reviewed by Bailey.

Early in the book the very important *Markov property* of a stochastic process is introduced. This property remains the basic postulate around which most of the book is woven since the ensuing material is concerned, with some exceptions, with successive generalizations of various models possessing the Markov property. In oversimplified terms, a stochastic process is said to possess the Markov property if the future behavior of the system is uniquely determined by, at most, the present state of the system. That is, the means by which the process reached its present state is immaterial to its future development. The transmission of genetic material from generation to generation in man, for example, is a system which can be said to have the Markov property since the "state" (i.e., genetic composition) of an offspring depends only on the "state" of the parents and not on the set of circumstances by which the parents arrived at that "state." Conversely, the passage of an individual through the various stages of health (or disease) in his lifetime may be non-Markovian, since the next state in which he finds himself could easily depend on his remote disease history as well as his present condition. Stochastic models which possess the Markov property and which allow transition from one state to another at regular intervals of time only are called *Markov chains*. Those which have the Markov property but which allow transitions to take place at any point continuously in time are called *Markov processes*. Bailey develops the important properties of this type of stochastic process and illustrates its usefulness in the physical sciences extensively.

Among the stochastic models discussed by Bailey are the discrete branching process (e.g., nuclear chain reactions, dissemination of a family name, passage of a mutant gene), the birth and death process and its generalizations (e.g., the growth of populations, the evolution of species, etc.), queueing processes (e.g., telephone line loadings, demand for hospital beds), epidemic processes (including the classical work by Kermack and McKendrick and the Reed-Frost model), and models for species competition and predication.

Each of these applications of the mathematics of stochastic processes is of considerable interest in an area of research in the physical sciences. It is worth emphasizing, however, that the usefulness of the techniques in Bailey's book is not restricted to investigations in the physical sciences, even though the applications have in the past been most frequent in that field. It does not take a great deal of translation, for example, to reinterpret an infectious agent as a rumor in the theory of epidemics, or to convert a study of change in health status to a study of mobility in social class. As in any mathematical theory, the names of the variables do not matter; rather it is the relationship between the variables that is the determining factor, and it is this *that* with which Bailey is concerned.

It is the opinion of the reviewer that this very fine book will, along with the rapidly growing body of literature on the application of stochastic processes,[1] prove extremely useful to those interested in furthering the quantification of almost any field. It will be especially useful to those in the physical sciences.

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REFERENCE

[1] Cramer, H., Model Building with the Aid of Stochastic Processes, *Technometrics,* 6, 133–159, 1964.