LIFE-TABLE FUNCTIONS FOR EGYPT BASED ON MODEL LIFE-TABLES AND QUASI-STABLE POPULATION THEORY

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INTRODUCTION

HE census reports on age are usually highly defective in a majority of the underdeveloped countries. Corrections and adjustments frequently are necessary to get a distribution representing as nearly as possible the real age structure of the population.

As pointed out by Lotka, a population closed to migration and influenced by a relatively constant age schedules of fertility and mortality will attain, after a considerable number of years, a fixed rate of growth and a fixed age distribution and the population is said to be "stable."

Improvements in public health during the last decades, have caused a marked decline of mortality in many countries. A population with a constant fertility and a smoothly declining death rate may be termed a "quasi-stable" population, the age distribution at any moment is nearly identical to the stable with the given constant fertility and the current mortality schedule. This is almost the case in many of the underdeveloped countries at the present time.

The purpose of this paper is to utilize the properties of the quasistable population to derive a life table and the related vital rates for Egypt, based on the reported age distribution of the last census and the rate of natural growth in the last intercensal period. There is no need for correcting and smoothing the reported age distribution that we have to start with. The smoothed age distribution will follow as a result of the life table constructed.

The Model Life-Tables of the United Nations 1955, (10), make it possible to derive the survival rates at the different age groups for Egypt's population consistent with the average levels of human mortality as observed in the various countries of the world.

Survival rates and life expectancies so obtained are compared with the similar results given previously by other techniques. The com-

¹ Statistical Department, Ministry of Education, Egypt, U.A.R. This study was made while the author was at the Office of Population Research, Princeton University during the academic year 1959–1960. A letter received from the Author, February 7, 1961, indicated that he had recently moved to Tripoli to join the teaching staff of the Faculty of Science, Libya University.

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parison shows that the previous life tables gave very high estimates for the survival rates and life expectancies to the old age-groups. It is then shown that these high survival rates were higher than in other countries known to have the highest life expectancy in the world.

The related points of the stable population theory and the effect of changes of fertility and mortality on the age structure of the population are reviewed to the extent required for explaining the calculation of the life table functions.

THE STABLE POPULATION

For the human population, Lotka (6) has shown that with constant age schedules of mortality and fertility, the population will ultimately have a constant rate of growth and a fixed age distribution. The proportion having age a in the stable population is given by

$$C(a) = be^{-ra}p(a)$$
^[1]

where

- C(a) = the density of age distribution such that, out of a total number N of persons, a number of NC(a)da are comprised within the age limits a and a + da
 - b = birth rate per head per annum
 - r = the rate of natural increase or the excess of birth rate b over death rate d i.e., r = b - d
- and p(a) = the probability of surviving a years after birth which equals the life table number of survivors la at age a divided by the size of the cohort at birth l_o.

Usually, the age distribution is given by the proportion in an age interval a to a + n say, and therefore

$$C(a, a + n) = \int_{a}^{a+n} b e^{-ra} \frac{l_{a}}{l_{o}} da \qquad [2]$$

$$= b e^{-r(a+\frac{1}{2}n)} L_{a}/l_{o}$$

The proportions in all the age groups will add up to l. Therefore

$$l = \int_0^w b e^{-ra} \frac{l_a}{l_o} da$$

from which the birth rate of the stable population is

$$b = l_o / \int_0^w e^{-ra} l_a da$$

$$= l_o / \sum_{a=0}^w e^{-r(a+\frac{1}{2}n)_n} L_a$$
[3]

and consequently, the proportion of the population in the age interval a to a + n is given by

$$C(a, a + n) = e^{-r(a+\frac{1}{2}n)} L_a / \sum_{a=0}^{w} e^{-r(a+\frac{1}{2}n)} L_a$$
 [4]

where

 ${}_nL_a = \text{the life table proportion in a given age group}$ and $l_o = \text{the radix of the life table.}$

The second fundamental equation for the stable population is that the birth rate

 $b = \int C(a)m(a)da$, limits of integration over the fertility age range which by substituting for C(a) becomes

$$l = \int_0^w e^{-ra} p(a)m(a)da$$
 [5]

where

m(a) = the maternity frequency per annum of females of age a.

This equation is an implicit expression for the stable rate of growth r. Solving for the real root of r satisfying this equation, Dublin and Lotka (6) found that, r is the root of the quadratic equation $\frac{1}{2}\beta r^2 + \alpha r - \log_e R_o = o$ given by

$$\mathbf{r} = \frac{1}{\beta} \left\{ -\alpha + \sqrt{\alpha^2 + 2\beta \log_e \mathbf{R}_o} \right\}$$
 [6]

where

$$\alpha = R_1/R_o$$
 and $\beta = \left(\frac{R_1}{R_o}\right)^2 - \frac{R_2}{R_o}$

and Ro, R1, R2 are given by

$$R_n = \int_0^w a^n p(a) m(a) da \quad \text{for } n = 0, 1, 2$$

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EFFECT OF MORTALITY CHANGE ON THE STABLE AGE DISTRIBUTION

Coale (3, 4) has shown that the decline of mortality—with a higher life expectancy—has a relatively minor effect on the stable age dis-



Fig. 1. Hypothetical curves indicating the effect of mortality change on the stable age distribution.

tribution provided that levels of fertility are unchanged. A more favorable mortality schedule -with a higher life expectancy-will yield a stable population with: (a) a higher proportion under 15, (b) a distribution with a lower average age, and (c) a slight increase in the fractions of the ages over 60, see Figure 1 for illustration and Figures 8 and 9 for Egypt's population.

Now let us consider a population in which fertility remains constant and mortality declines. This is the case in a majority of the populations of underdeveloped countries at the present time. Let us imagine that at a given moment mortality ceases to decline and remains constant at the level attained in that moment. After a certain time the population will attain the stable population structure. However, at that moment, as pointed out by Bourgeois-Pichat (2), the population is already very close to this stable form. It follows that the population in which fertility remains constant and mortality declines, may be considered similar at each moment to a stable population. In other words, in these populations, there is at each moment between the age composition, the birth rate, the mortality rate and the rate of increase approximately the same relations as in a stable population.

EFFECT OF FERTILITY CHANGE ON THE STABLE AGE DISTRIBUTION

The role of fertility in shaping the stable age distribution is much more effective than that of mortality. This is known at least within the prevailing levels of mortality and fertility observed in the World. Differences in fertility produce quite different stable age distributions. This, combined with the fact that changes in mortality only slightly

affect the age structure, means that a schedule of fertility by age is sufficient to give a fair approximation to the stable age distribution even if mortality rates by age are not known.

If a high fertility age distribution is compared with a low fertility distribution with the same mortality, the higher fertility population will have higher proportions at younger ages, the two



Fig. 2. Hypothetical curves indicating effect of fertility change on the stable age distribution.

distributions will tend to intersect at the mean of the average ages, Figure 2.

The heavy influence of fertility on the stable age distribution is due to the fact that the two terms b and $\int_{15}^{49} C(a)da$ which define the general fertility rate (the general fertility rate = no. of live births/no. of women aged 15-49) are respectively the ordinate of the age proportion distribution curve at its origin, given by C(o) = b, and the area under the middle part of the curve, given by $\int_{15}^{49} C(a)da$. Thus if the general fertility rate is increased, the starting point of the age curve will rise by almost the same proportion but the change in the middle area of the curve will be comparatively small. This means that we shall have higher proportions at the younger ages. In general the approximate form of the age distribution is determined by the level of fertility. The level of mortality has more or less second order effects on the distribution.

PATTERNS OF MORTALITY AND MODEL LIFE-TABLES

The typical variation of mortality with advance in age during the life span is represented by a "u" shaped curve which starts high at birth, declines rapidly to a minimum around age 12, and then increases slowly through adolescence and maturity and finally rises sharply at old ages.

Improvements in conditions of living and standards of health are reflected in the improvements in mortality rates which change from high to low levels. Even at the generally low levels of mortality the variations by age are still characterized by the "u" shaped pattern.

The patterns of transition from high to low mortality levels during the last fifty years have been studied by the Population Branch of the Bureau of Social Affairs of the United Nations (10), Series A/22. The study covered 158 selected national life tables of 50 different countries of all the continents. For every age group there was a maximum and a minimum value of all the different specific mortality rates at that age. The ratio between maximum and minimum mortalities becomes smaller and smaller as we pass from younger to older agegroups. That is, the variations between mortality rates in the World are less discernible in the older ages whereas younger ages are more sensitive to changes in mortality levels. Consequently, an appropriate measure of the general transition from high to low mortality must be sought in the lower age brackets, where the variations of mortality are relatively large.

For every pair of consecutive age-groups, the scatter points of the world mortality rates ${}_{5}q_{x}$ and ${}_{5}q_{x+5}$ of the two age-groups are plotted and a second-degree parabola of the form

$${}_{5}q_{x+5} = A + B {}_{5}q_{x} + C {}_{5}q_{x^{2}}$$
 for $x = 0, (5), 75$

is fitted. Also a similar parabola is fitted to q_0 and ${}_5q_0$. Therefore it will be sufficient to know the level of infant mortality q_0 (or the level of mortality at a young age-group), then all of the mortality rates at the different age groups are estimated from these equations. The estimates are at best possible averages of the World's mortality rates. For forty equally spaced levels of infant mortality q_0 , the Model Life-Tables mortality rates ${}_5q_x$ (for x = 0, (5), 80) were given in Series A/22, (10). The other functions of the Model Life-Tables were tabulated in Series A/25, (11).

THE GROWTH OF POPULATION IN EGYPT

According to the 1897 census, Egypt's population was then 9.734 millions. Decennial censuses afterwards show a gradual increase in the population. The census of 1947 indicated a population of 18.967 millions. That is, the population has nearly doubled in 50 years. It

is found that there was overcounting in the 1947 census. An adjustment for overenumeration given by El-Badry (7) reduced it to 17.907 millions in 1947. In plotting the population size in the different

census years, Table 1 and Figure 3, we see that the population growth since 1897 is almost constant as the census points form a line which is nearly straight. The 1947 returns, however, show a higher rate of increase than that prevailing in the previous censuses and the adjustment for overenumeration was made



Fig. 3. Population of Eygpt, 1800 to 1957.

by the continuation of the 1907–1937 rate of increase. In 1957, the preenumeration stages were conducted, but because of the unfavorable conditions that occurred in that year the census was postponed. It was conducted in 1960 for the new United Arab Republic, the north and the south regions (Syria and Egypt). According to estimates from the National Sample Survey of Labor Force in Egypt, the 1957 population was 23.632 millions. This estimate shows a higher rate of population growth in the last decade than that during 1907-1937 or 1907-1947. Patterns of birth and death rates in Egypt during the last years will indicate, as shown later, an increase in the rate of growth during the last decade. The rates of growth r per head per annum during the

Census Year	Males	Females	Total
1897	4,948	4,787	9,734
1907	5,617	5,573	11,190
1917	6,369	6,349	12,718
1927	7,058	7,120	14,178
1937	7,967	7,954	15,921
1947	9,392	9,575	18,967
1947 a	8,951	8,956	17,907
1957 ^ь	11,789	11,843	23,632

Table 1. Population of Egypt (in thousands).

Estimation of El-Badry (7) for correcting overenumeration.
 Estimate from The National Labor Force Sample Survey.

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intercensal period t years are calculated by the exponential law

$$r = \frac{l}{t} \log_{\circ} (P_t/P_o)$$

$$= 2.302585 \times \frac{l}{t} \log (P_t/P_o)$$
[7]

where P_o is the population size at the start of the interval t and P_t is its size by the end of that interval.

Population Rate of Growth (r) per head per year, for Egypt

Decade	Males	Females	Males and Females
1897-1907	.012681	.015203	.013939
1907-1917	.012566	.013036	.012800
1917–1927	.010274	.011461	.010865
1927–1937	.012116	.011076	.011594
1937–1947	.011645	.011864	.011755

The tabulated rate of growth for the 1937–1947 decade is for the adjusted value of 1947 census. If the reported value of the 1947 census is considered, the rate of growth in 1937–1947 is .017505 (for both sexes). Also, in the decade 1947–1957, the rate is found to be .021989 (for both sexes). Thus during the last one or two decades, it appears that the rate of growth has been rising above the rate that had remained almost the same between 1897 and 1947, when it was between .011 and .014 per head per year. From 1947 until 1960 no full census was taken. It is hoped that the 1960 census will provide a more reliable estimate of growth rate in the last decade.

BIRTH AND DEATH RATES IN EGYPT

Birth and death rates exhibit serious underregistration in almost all the low-income countries. Despite the fact that in Egypt registration of births and deaths has been compulsory by law for quite a number of years, there is nevertheless a high percentage of underregistration. The underregistration of births is most acute in villages located far from the city or from the health bureau area. The decree for registering births, issued in 1912, imposes the responsibility for reporting on the mother, father, or any relative in the same household. It also imposes responsibility on the doctor or nurse who managed the delivery. Birth registration should be within fifteen days after the birth day. For deaths, notification is the responsibility of relatives or any member of the household. A death should be reported by the doctor or health representative or by the "Omda" or "Sheikhel-balad" who are representatives of the area. Registrations of births and deaths usually are made in the public health offices. In remote areas registration is made in the tax collector's office (called Sarraf). The public health offices send a weekly report of these events to the Statistical Department; the tax collectors' offices send their reports monthly. It is essential for every person in the country to submit a birth certificate for employment, civil service, identification, army service and compulsory school enrollment. Persons that escaped birth registration will need to register themselves on entering school or employment.

An estimate of the percentage of underregistration of births based on a comparison of the crude birth rates in the health bureau areas with that in the whole country is found to be about 4 per cent during

A-Average Rates in 5 Year Periods, from 1906 to 1954										
	All	Areas	Health Bureau Areas							
Period	Birth Rate (Average)	Death Rate (Average)	Birth Rate (Average)	Death Rate (Average)						
1906-1909	43.0	25.5	44.2							
1910-1914	42.5	26.2	45.2							
1915-1919	39.8	31.3	40.8	41.0						
1920-1924	42.8	25.8	48.7	32.6						
1925-1929	43.9	26.5	45.8	32.8						
1930-1934	43.7	27.0	45.2	28.8						
1935-1939	42.8	26.9	45.0	29.6						
1940-1944	39.6	26.8								
1945-1949	42.4	23.0								
1950-1954	43.9	18.7								

Table 2. Crude birth and death rates per 1,000 of population in Egypt.

B-SINGLE YEAR RATES, FROM 1935 TO 1955

Year	Birth Rate	Death Rate	Year	Birth Rate	Death Rate	Year	Birth Rate	Death Rate
1935	41.3	26.4	1942	37.6	28.3	1949	41.8	20.6
1936	44.2	28.8	1943	38.7	27.7	1950	44.4	19.1
1937	43.4	27.1	1944	39.8	26.0	1951	44.8	19.3
1938	43.2	26.3	1945	42.7	27.7	1952	45.1	17.7
1939	42.0	25.9	1946	41.2	25.0	1953	42.5	19.5
1940	41.3	26.3	1947	43.8	21.4	1954	42.5	17.8
1941	40.4	25.7	1948	42.7	20.4	1955	40.0	17.6

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the period 1925 to 1940. Underregistration can also be estimated by projecting the number of children under 5 years of age in the census to births by the division ${}_{5}L_{o}/{}_{5}l_{o}$ (the probability of surviving from



Fig. 4. Birth rates and death rates in Egypt (five year averages), 1906–1954.

birth to ages zero to 5) and comparing the projected result with the registered number of births in the five years before the census. This method will give, using the 1947 census data, an estimate of about 7 per cent for birth underregistration.

Variations of birth rates in Egypt were not wide over a long period of time. As seen in Table 2 and Figure 4. the registered crude birth rate averages between 42 and 44 births per thousand of population. The nearly fixed rate extends from 1906 to 1954, with the exception of the two periods 1915-1919 and 1940-1944 which reflect the effects of the World Wars I and II. The annual variations, as seen

in the last twenty years, never diverge from this fixed birth rate by more than 2 per thousand.

The annual death rate during the twenty years preceding 1945 remained almost constant up to 1945, according to the reported figures. (These figures were considerably underreported, especially in the earlier years, as will be seen later.) The average death rate from 1935 to 1945 was 27 per thousand of population. The annual variations from this average do not exceed 2 per thousand. After

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1945 a clear decline of death rate took place at the rate of about one per thousand per annum.

THE CENSUS AGE DISTRIBUTION

Census age reports in Egypt are, as in many other countries, full of errors. Heapings at ages ending in 0, 5, and some even numbers, overreporting of old ages, underreporting of children, underestimation of females in certain age groups as 40-49, etc., are typical distortions. Other types of errors are quite frequent, and the data frequently have been adjusted and smoothed before being used for demographic analysis.

Different methods have been followed for age-distribution smoothing. In most cases smoothing by fitting a frequency distribution curve was followed. Kiser (9) and El-Shanawany (8) in constructing life tables for Egypt fitted the Pearson Type IX frequency curve of the form

$$\mathbf{y} = \mathbf{y}_{\mathbf{o}}(\mathbf{a} - \mathbf{x})^{\mathbf{m}}$$
 [8]

to the age distributions. For an Indian life-table a Type I frequency curve was applied of the form

$$y = y_o \left(1 + \frac{x}{a_1}\right)^{m_1} \left(1 - \frac{x}{a_2}\right)^{m_2}$$
 [9]

where the age span w is $a_1 + a_2$, and the origin is at the mode.

Before smoothing, adjustments of the irregularities in some parts of the age distribution are often made. These methods, although clearly improving the raw data, are laborious and inevitably reflect the personal judgment of the author.

The method applied here for adjusting the reported age distribution is based on the following characteristics of the population:

- 1. Fertility can be considered as practically constant.
- 2. Mortality has varied in recent years smoothly, and in accordance with the
- typical age-patterns of mortality prevailing in the world.
 - 3. Absence or negligible importance of international migration.

These basic conditions already characterize the population of Egypt as well as many of the other low-income countries. Unchanging fertility and fixed mortality will lead to the stability of the population with ultimately fixed age structure. Changes in mortality will introduce only slight changes in the stable age structure and the population <u>م</u> ا

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		Males		Females		
	1947	1937	1927	1947	1937	1927
0-4	13.6	13.0	14.1	13.6	13.7	14.5
5-9	12.9	13.9	13.3	12.4	13.8	12.9
10-14	12.2	12.9	12.2	11.2	11.0	10.1
15-19	10.5	8.9	9.6	9.6	8.0	8.6
20-24	7.2	6.8	7.4	7.4	7.1	8.1
25–29	7.3	7.7	8.1	8.2	8.7	9.2
3034	6.6	7.0	7.0	7.2	8.0	8.0
3539	7.0	7.5	7.0	6.8	6.8	6.2
40-44	6.1	6.0	5.4	5.9	5.9	5.8
45-49	4.6	4.3	4.0	4.3	3.9	3.4
50-54	4.5	4.1	3.8	4.7	4.2	4.3
55–59	1.8	1.8	1.7	1.8	1.7	1.5
60+	5.5	5.8	6.1	6.5	6.9	7.0
Not Given	0.3	0.2	0.3	0.3	0.2	0.3
Total	100.1	99.9	100.0	99.9	99.9	99.5

Table 3. Reported age distribution (in percentages of total population) in the census years 1927, 1937 and 1947.

will be approximately stable, i.e. "quasi-stable." The similarity of the reported age distributions of the 1927, 1937 and 1947 censuses is seen in Table 3 and Figure 5.

Fig. 5. Percentage age distributions, by sex, as reported in the censuses of Egypt 1927-1947.



FINDING THE LIFE TABLE FUNCTIONS

The reported age distribution of the census of 1947 and the rate of growth (r) in the period between the last two censuses

(r = .011645 for males and r = .011864 for females)

are assumed as characteristics of a "quasi-stable" population.

Recalling the first fundamental formula of stable population, we get

$$C(a)e^{ra} = bl_a/l_o$$
 [10]

If $\frac{n(a)}{N}$ is the reported proportion of population in the age-group whose mid-point is a, then $\frac{n(a)}{N} e^{ra}$ will be proportional to l_a . The values of l_a so obtained from the census data will be subject to all the irregularities and errors of age reporting; we shall call it " l_a reported" to distinguish it from the life table l_a which represents the survivors at age a.

An average l_a value covering all ages or at least from age five or ten and over (in which case we avoid the usually most erroneous group of under five years of age) will be fairly reliable. The errors of age reporting will to a large degree neutralize each other as the heapings in certain ages are usually at the expense of the neighboring ages. The person-years in the life table above age 10 (T₁₀) as calculated from the "l_a reported" values will approximate fairly well the value that would be calculated from an actual life table.

Now, to estimate the expectation of life at age ten, \mathring{e}_{10} , from the " l_a reported" values, we have:

estimate of
$$\mathring{e}_{10} = \frac{l}{l_{10}} \int_{10}^{w} l_{x} dx = \frac{T_{10}}{l_{10}}$$
 [11]

where the l's are the reported values given by the census age distribution.

To evaluate T_{10} , the average between two successive reported l_x values is multiplied by the length of the age group and then summed for all age groups from 10 and over. For the last age group, 85 years and over, approximation is given by

 $L_{85+} = l_{85} \log_{10} l_{85}$ (From Manual III, U.N. p. 23).

More specifically, when the reported age distribution is in 5-years

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age-groups, we shall have the set of values of reported $l_{2.5},\,l_{7.5},\,\ldots\,,$ $l_{82.5}$ and $l_{87.5}.$ Therefore

$$l_{10} = \frac{1}{2} \left(l_{7.5} + l_{12.5} \right)$$
 [12]

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$$T_{10} = 5[.25(l_{7.5} - l_{12.5}) + \sum_{a=10}^{80} l_{a+2.5} - .25(l_{82.5} - l_{87.5})] + L_{85+}$$
[13]

The estimates of expectation of life at age 10, from the 1947 data, give the following results (see Table 4):

for males: estimate of
$$\mathring{e}_{10} = 38.732$$
 years

for females: estimate of
$$\mathring{e}_{10} = 42.006$$
 years

With these estimates of \mathring{e}_{10} , we get the life table function l_x , which

Table 4. Reported age proportions $\frac{n(a)}{N}$ and reported survivors (l_a reported) for the 1947 Census.

	Males				Females	
Pivotal Age (a)	$\frac{n(a)}{N}$	e ^{.011645a}	la	$\frac{n(a)}{N}$	e ^{.011864a}	l _a
	(1)	(2)	(1) × (2)	(1)	(2)	(1) × (2)
2.5	.1362	1.0295	. 1403	.1363	1.0301	.1404
7.5	.1287	1.0913	.1405	.1244	1.0931	.1360
12.5	.1216	1.1567	.1407	.1119	1.1599	.1298
17.5	.1048	1.2260	.1285	.0958	1.2307	.1179
22.5	.0722	1.2995	.0938	.0737	1.3060	.0963
27.5	.0730	1.3775	.1006	.0821	1.3858	.1138
32.5	.0660	1.4600	.0964	.0720	1.4705	.1059
37.5	.0702	1.5476	.1086	.0683	1.5603	.1065
42.5	.0606	1.6404	.0994	.0591	1.6557	.0979
47.5	.0456	1.7387	.0793	.0434	1.7569	.0762
52.5	.0449	1.8429	.0827	.0468	1.8643	.0873
57.5	.0182	1.9534	.0356	.0181	1.9782	.0358
62.5	.0268	2.0706	.0556	.0312	2.0991	.0655
67.5	.0089	2.1947	.0196	.0086	2.2274	.0191
72.5	.0115	2.3263	.0267	.0143	2.3635	.0338
77.5	.0025	2.4657	.0062	.0025	2.5079	.0063
82.5	.0037	2.6136	.0097	.0055	2.6612	.0146
87.5	.0018	2.7703	.0050	.0025	2.8239	.0070
I	Estimate of	$\hat{e}_{10} = 38.7$		Estima	te of $\hat{e}_{10} = 4$	42.0

is the survivors to age x out of a cohort of 100,000 at birth, by interpolation from the Model Life Tables in Manual III U.N. Series A/25, (11). Before interpolating, we must get the tabular values of \mathring{e}_{10} between which our estimates lie. These are given by

$$\mathring{e}_{10} = \frac{l}{l_{10}} \left(T_o - {}_{10} L_o \right)$$
[14]

where l_{10} , T_0 and l_0L_0 are Model Life-Table values.

The numerical tabular values of \mathring{e}_{10} corresponding to the tabular levels of \mathring{e}_0 are as follows

Model Life-Table values of \mathring{e}_{10}				
Μ	\mathbf{F}			
27.4816	37.3924			
39.1012	39.2709			
40.7370	41.1702			
42.4373	42.9376			
	Model Life-Tab M 27.4816 39.1012 40.7370 42.4373			

The other functions are calculated by the following simple relations:

(i) The survivors in the age group x to x + 5 are derived by interpolation from the ${}_{5}L_{x}$ table or by calculation from

$${}_{5}L_{x} = 2.5(l_{x} + l_{x+5}) \text{ for age groups 5-9 to 80-84} \\ L_{o} = .25l_{o} + .75l_{1} \text{ for the first year of life} \\ {}_{4}L_{1} = 1.9l_{1} + 2.1l_{5} \text{ for age group 1-4} \\ {}_{5}L_{o} = .25l_{o} + 2.65l_{1} + 2.1l_{5} \text{ for age group 0-4} }$$

$$[15]$$

(ii) ${}_{5}q_{x} = \frac{l}{l_{x}} (l_{x} - l_{x+5}) = \text{probability of dying between} \\ age x and x + 5 \\ {}_{5}p_{x} = l - {}_{5}q_{x} = \text{probability of surviving from} \\ age x to age x + 5 \end{bmatrix}$ [16]

(iii)
$$T_x = \sum_{x}^{w} {}_{5}L_x = \text{total number of years of life remain-ing to survivors at age x}$$

and $\hat{e}_x = \frac{T_x}{l_x} = \text{average number of years remaining}$ to survivors at age x

and thus the life table functions are calculated as given in Table 5.

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Life-Table Functions for Egypt

Age	Males			Females			
x	l _x	5 P x	ê _x	lx	δPx	ê _x	
0	100,000	.6155	29.0	100,000	.6784	34.3	
5	61,561	.9450	41.5	67,834	.9538	45.0	
10	58,174	.9646	38.7	64.701	.9670	42.0	
15	56,105	.9514	35.1	62,563	.9541	38.4	
20	53,379	.9351	31.8	59,690	.9396	35.1	
25	49,913	.9276	28.8	56,082	.9315	32.2	
30	46,297	.9179	25.8	52,239	.9253	29.4	
35	42,494	.9035	22.9	48,335	.9187	26.6	
40	38,395	.8817	20.1	44,404	.9119	23.7	
45	33,851	.8547	17.5	40,493	. 8986	20.7	
50	28,932	.8239	15.0	36,387	.8770	17.8	
55	23,837	.7820	12.7	31,910	.8458	14.9	
60	18,641	.7294	10.5	26,988	.7931	12.2	
65	13,597	.6540	8.5	21,403	.7200	9.7	
70	8,893	.5440	6.7	15,411	.6128	7.5	
75	4,838	.4161	5.1	9,444	.4758	5.7	
80	2,013	.2573	3.8	4,493	.3181	4.3	
85	518		2.7	1,429		3.2	

Table 5. Life table for Egypt in the period 1937-1947.

THE ADJUSTED AGE DISTRIBUTION

Unlike the usual procedures of correcting the census reported data and then constructing the life table, the process here is reversed by constructing the life table first and from the life table functions the adjusted age distribution is derived. The proportion of the population in the age group a to a + n is, on the basis of stable age distribution, given by equation [4].

$$C(a, a + n) = e^{-r(a+\frac{3}{2}n)} L_a / \sum_{a=0}^{w} e^{-r(a+\frac{3}{2}n)} L_a$$

This procedure was followed in calculating the proportional age distribution in Table 6. Figures 6 and 7 show how adequately this procedure gives smoothed age distributions fitting the reported distributions of the census and correcting its errors.

INTRINSIC VITAL RATES

The birth rate associated with the quasi-stable population is given by equation [3]

$$b = l_o / \sum_{a=o}^{w} e^{-r(a+\frac{1}{2}n)} L_a$$

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Pivotal		MA	LES			Fem.	ALES	
Age	e ^{011645a}	sLa/lo	(1) × (2)	C(a)	e ^{011864a}	sLs/lo	(1) X (2)	C(a)
(a)	(1)	(2)			(1)	(2)		
2.5	.9713	3.5056	3.4050	.158	.9708	3.7660	3.6560	.148
7.5	.9164	2.9934	2.7431	.127	.9149	3.3134	3.0313	.122
12.5	.8645	2.8570	2.4700	.115	.8622	3.1816	2.7431	.111
17.5	.8156	2.7371	2.2325	.104	.8125	3.0563	2.4833	.100
22.5	.7695	2,5823	1.9871	.092	.7657	2.8943	2.2162	.089
27.5	.7260	2.4052	1.7461	.081	.7216	2,7080	1.9541	.079
32.5	.6849	2.2198	1.5203	.071	.6801	2.5143	1.7099	.069
37.5	.6462	2.0222	1.3067	.061	.6409	2.3185	1.4859	.060
42.5	.6096	1.8061	1.1011	.051	.6040	2.1224	1.2819	.052
47.5	.5751	1.5696	.9027	.042	. 5692	1.9220	1.0940	.044
52.5	.5426	1.3192	.7158	.033	.5364	1.7074	.9159	.037
57.5	.5119	1.0620	.5436	.025	. 5055	1.4724	.7443	.030
62.5	.4830	. 8059	.3892	.018	.4764	1.2098	.5763	.023
67.5	.4556	. 5623	.2562	.012	.4490	.9203	.4132	.017
72.5	.4299	. 3433	.1476	.007	.4231	.6214	.2629	.011
77.5	.4056	.1713	.0695	.003	.3987	.3484	.1389	.006
82.5	.3826	.0633	.0242	.001	.3758	.1480	.0556	.002
87.5	.3610	.0141	.0051	.000	.3541	.0451	.0160	.001
Totals			21.5658	1.000	•		24.7788	1.000
Intrinsic birth r	ate:		.0464		Intrinsic b	irth rate	.0404	
Intrinsic death	rate:		.0347		Intrinsic d	eath rate	.0285	

Table 6. Intrinsic birth and death rates and the stable age proportions, C(a), of Egypt's population in 1947.

The intrinsic birth rate for males (b_m) and for females (b_f) calculated by this formula are given in Table 6 and are as follows:

 $b_m = 46.370$ boys born per 1000 of male population $b_f = 40.357$ girls " " " " female "

For the combined birth rate of males and females we may estimate the sex ratio. During the five years 1945–1949, there were 4,040,255 live births of whom 2,116,696 were males, therefore,

Sex ratio of females
$$=$$
 $\frac{\text{Female births}}{\text{Total births}}$
 $=$.4761

Ratio of male births to female births is

$$m:f = .5239/.4761 = 1.1004:1$$

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Fig. 6. Reported age distribution and quasi-stable age distribution of males in Egypt, Census 1947.

The combined birth rate b is the harmonic weighted average of the male and female birth rates and given by

$$b = (m + f) / \left(\frac{m}{b_m} + \frac{f}{b_f}\right)$$
[18]

which is found to be of the value

b = 43.620 children born per 1000 of population.

The corresponding intrinsic death rates as given by d = b - r have the following values

$$d_m = 34.7 \text{ per 1000 of male population}$$

$$d_t = 28.5 \quad \text{`` `` female ``}$$

$$d = 31.9 \quad \text{`` `` population}$$

and

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Fig. 7. Reported age distribution and quasi-stable age distribution of females in Egypt, Census 1947.

The intrinsic, or standardized, birth and death rates represent the ultimate, or long run average, of crude birth and death rates if the population remains subject to constant conditions of fertility and mortality. When this standardized situation prevails, the ultimate annual rate of growth (r) of the population will remain constant and the same for both sexes, and the age structure will be ultimately fixed. A comparison of the intrinsic rates with the crude rates between 1937 and 1947 is given in the following table

	CRUDE RATE PE	r 1,000 Populati	on in 1937–1946	INTRINSIC RATE
	Maximum	Minimum	Average	PER 1,000 POPULATION
Births Deaths	43.4 28.3	37.6 25.0	41.0 26.6	43.6 31.9

This comparison suggests that birth registration was fairly complete while death registration was far from complete. The underregistra(he

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Fig. 8. Chart showing slight effect of improved mortality on age structure of stable population of Egyptian males. Basic data from 1947 Census.

tion of births was about 6 per cent of the average crude birth rate while the underregistration of deaths was 20 per cent of the average crude death rate.

It is interesting to ascertain what the intrinsic birth and death rates would be if the population of Egypt in 1947 were 18.967 millions as given by the census reports. Also, how would the estimated age structure of the population be affected if the official figures were accepted as correct rather than as inflated?

If the population in 1947 were 18.967 millions, the rate of increase during the decade 1937-47 would be .01646 for males, .01855 for females and .01751 for both sexes. Following the same procedure for constructing the life table and the stable age distribution, it is found that the expectation of life at age ten will be

	$e_{10} =$	43.9	years	for	males
nd	$e_{10} =$	50.4	"	"	females

The stable age distribution for these levels of \mathring{e}_{10} , which are much higher than the 38.7 and 42.0 found before, give a typical illustration of the effect of a decline in mortality on the stable age structure. Figures 8 and 9 show how slight are the changes in the age structure of Egypt's population when mortality declines.



Fig. 9. Chart showing slight effect of improved mortality on age structure of stable population of Egyptian females. Basic data from 1947 Census.

The intrinsic vital rates are found to be:

b = 39.7 children born per 1,000 of population

d = 22.2 deaths per 1,000 of population.

Comparing these with the crude rates will show that: (a) the assumed increase of the population implies a very substantially lower level of mortality and a slightly lower level of fertility; (b) the low value of the intrinsic death rate in comparison with the crude rate (which is probably too low because of incomplete death registration) supports the view that the 1947 census was a relative overenumeration.

EVALUATION OF RESULTS

To judge the results given by the technique followed in this paper, the following comparisons were made:

1. Comparison of the intrinsic vital rates of birth and death with the actual reported rates.

2. Comparison of the stable age structure of the population with the reported age structure of the census.

3. Comparison of the life table functions of survival rates and life expectancies with those of life tables constructed by other methods. Comparisons (1) and (2) have been covered in the previous articles of the text and proved adequacy of results were obtained. For com-

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parison with the life tables constructed by previous authors, the following other life tables for Egypt were considered:

1. The "First National Life-Table for Egypt" for the period 1917-1927 constructed by El-Shanawany (8) in 1936. The method applied was based on smoothing the reported age distribution by a Pearson type IX curve and following cohorts in two censuses to get the survival rates.

2. Life-Table for the period 1927-1937 constructed by Kiser (9) in 1944 by adjusting² the census age distribution and smoothing by curve fitting. Similar in principle to the census differencing method used by El-Shanawany.

3. "The Egyptian National Life-Table No. 2" for the period 1936–1938 by Abdel-Rahman (1) in 1948 constructed by use of death reports and the census age distribution of the population to calculate the age-specific death rates.

4. Life-Table for the period 1907-1947 constructed by El-Badry (7) in 1955. The procedure was based on the stability of the census age distribution. According to the stable population theory, the proportion of the population aged a will be

$$C(a) = be^{-ra}l_a/l_o$$

where the terms have the same meanings as given before. From this equation the survival rate from age a to age a + n is deduced as follows

$$\frac{C(a + n)e^{r(a+n)}}{C(a)e^{ra}} = \frac{l_{a+n}}{l_a} = {}_np_a$$

Therefore, to calculate the life table functions, it was necessary to correct the reported age distribution of the census to get an estimate of C(a). A series of corrections was made of the census age frequencies, as correction of female frequencies under 20 years of age, correction of male frequencies in the first three age-groups, correction for underestimation of females at ages 40–59, and correction for the old age frequencies. Having corrected the frequencies of the five censuses

² Kiser's adjustment to the census age-groups reports:

etc.

0-4 No change except adjustment for underenumeration 5-9 Census figure plus one-fourth of age 10 10-14 Census figure less one-fourth of age 10 plus one-half of age 15 15-19 """one-half "" 15 """"20 20-24 """""20 """"20 " from 1907 to 1947 the average is taken as an estimate of the frequency C(a) at age a. Hence starting with the above equation of $_{n}p_{n}$ the life table was constructed.

It was necessary to give a brief hint on each of the methods used in order to reveal the reasons for any discrepancies.

The expectation of life e_x for a given age x is plotted for the different life tables in Figure 10.

The following points are notable:

1. The lines form a close bundle, a situation which conforms with the fact of stabilization of the population structure over the whole period 1907-1947 covering the periods of all the tables.

2. Crossing and recrossing of some lines that might be due to inconsistencies in the many corrections used for adjusting the age frequencies.

3. The line of the life table of the period 1937–1947 based on the quasi-stable population theory and Model Life-Tables places itself in conformity with the other lines, but it notably deviates from them at the older ages; a fact that will be briefly interpreted in the next section.

The survival rate of males for the life table of the period 1907-1947, which has a life expectancy at birth \mathring{e}_0 between 31.8 as maximum and 28.5 years as minimum was plotted, Figure 11, with the two Model Life-Tables of $\mathring{e}_0 = 27.5$ and $\mathring{e}_0 = 32.5$ to envelope the first line. Crossing and recrossing and high deviation at the older ages, of 60 years and over, can again be noted. The same remarks also apply to the females.

The higher survivor rate and expectation of life at the older agegroups can be explained in part at least on age overreporting in these groups. El-Badry says: "it was preferred to use the tail of the 1947 distribution from age 60 upward as the tail of the average distribution in order to utilize what is apparently a systematic improvement in the overestimation of old ages from one census to the next," which overlooks the existence of overestimation in the 1947 census itself.

OVERESTIMATION OF OLD AGE-GROUPS

Census records seem clearly to overestimate the old age-groups in Egypt. The tail of the 1947 census age distribution was analyzed and compared with other countries of higher life expectancy and longer span of life than Egypt and with countries of similar situation to Egypt's population. As age reporting is highly correlated with the literacy status of the population, it seemed desirable to consider some





Fig. 11. Survival rates $({}_5P_x)$ by sex and age in Egypt, as computed in indicated life tables.

countries similar to Egypt in this respect. Moreover two populations that differ in educational and social conditions but in the same country were considered. Censuses dated near to 1947 of the chosen countries were taken.

In Egypt's census, those of age 70 years and over were only 2.2 per cent of the total population as compared with 4.8 per cent in the United States, 6.9 per cent in England and Wales, 6.4 per cent in Sweden, 3.2 per cent in Japan, 2.2 per cent in Turkey and 2.1 per cent in Ceylon (see Table 7). With these percentages in mind we find that

Census Year	1950	1951	1950	1945	1947	1945	1946
Age	U.S.	England and Wales	Sweden	Japan	Egypt	Turkey	Ceylon
100+ 90+ 80+ 70+	.06 2.04 23.45 100	.01 1.34 21.50 100	.01 1.82 23.69 100	.01 .94 17.67 100	1.22 7.21 30.52 100	1.07 7.21 30.02 100	.36 4.74 28.34 100
70+ Per Cent Total Pop.	4.83	6.85	6.41	3.17	2.22	2.15	2.05

Table 7. The percentages of population aged 80+, 90+ and 100+ relative to 70+ for some selected countries in the stated censuses.

those aged 90 years and over are 7.2 per cent in Egypt (of the tail 70+) as compared with only 2.0 per cent in United States. 1.3 per cent in England and Wales, 1.8 per cent in Sweden, .9 per cent in

Fig. 12. Survival rates ($_{5}P_{x}$) at ages 60 and over for selected countries. (See Table 8.)



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Country	Years	Expec- tation of Life	SURVIVAL RATES 6PX BY AGE GROUPS							
		at Birth ê0	60 64	65 69	70– 74	75–79	8084	85-89	9095	
United States	1950	68.4	.8907	.8476	.7758	.6774	. 5701			
England and Wales	1950	68.9	. 8964	.8424	,7593	.6329	.4763	1		
Sweden	1946-1950	70.3	.9159	.8657	.7834	.6578	.4896			
Japan	1947	52.0	.8461	.7770	.6840	.5634	.4172			
Ceylon	1945-1947	45.8	.8479	.7842	.7013	. 5834	.3976			
-										
Egypt (Males)	1907-1947	28.5	.7880	.7488	.6796	.6043	.5848	. 5483	.5441	
,		31.8								
Egypt (Fem.)	1907-1947	30.2	.8144	.7795	.7082	.6394	.6265	. 5915	.5448	
		33.6								

Table 8. Life expectancy \hat{e}_0 and life table survival rate $_{\delta}p_x$ to old age groups (both sexes) for some selected countries compared with that in Egypt.

Japan and, as in Egypt, it is 7.2 per cent in Turkey and 4.7 per cent in Ceylon. See Table 7 for further similar comparisons.

As a consequence of this overheaping at old ages in Egypt, its survival rate line, Figure 12, crosses nearly transversally the survival rate lines of the countries whose life expectancies are far higher than Egypt. (See Table 8).

Undoubtedly there is systematic improvement from one census to the next which serves to reduce this overheaping as the literacy and health status of the population are improved. This is the case not only in Egypt but also in other populations. In the United States, the five censuses from 1910 to 1950 show the clear evidence of overheaping in old ages of the non-whites. In the 1910 census, for example, the percentages of 70+ to total population were 2.56 per cent for the whites and only 1.74 per cent for the non-whites, while we see that those of age 90+ in the same census are only 1.57 per cent (of the tail 70+) for the whites and 6.43 per cent for the non-whites. Table 9 shows clearly the overestimation in old ages for the non-whites compared with the white population and how this erroneous population phenomenon is systematically improved from census to census.

Accordingly, the low survival rate (or the expectation of life) for old ages presented by the 1937–1947 life table based on Model-Life Tables and quasi-stable theory seems to be a reasonable estimate for Egypt's population, while the low mortality in the other Egyptian life tables probably has its source in systematic relative overcounts

CENSUS YEAR	1910		1920		1930		1940		1950	
Age	White	Non- White								
100+	0.04	1.57	0.04	1.50	0.03	1.13	0.00	0.81	0.00	0.66
90+	1.57	6.43	1.56	5.80	1.48	5.24	1.46	4.47	1.78	3.63
80+	20.97	27.97	20.99	26.45	20.29	27.45	21.60	23.58	23.37	23.10
70+ 1	100	100	100	100	100	100	100	100	100	100
70+ Per Cent Total Pop.	2.56	1.74	2.80	1.87	3.32	1.83	4.12	2.46	5.05	3.03

Table 9. Percentages of white and non-white population of United States aged 80+, 90+ and 100+ relative to 70+ in censuses 1910–1950.

at the older ages. Of course, the higher expectancies at old ages given by the life tables mentioned might express an exceptional character of Egypt's population, but this is very unlikely.

Acknowledgment

The writer acknowledges his indebtedness to Professor Ansley Coale, Director, Office of Population Research, Princeton University, for his encouragement, discussions, and comments in the preparation and presentation of this work.

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