MORTALITY DECLINES AND AGE DISTRIBUTION[1]

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I

RECENT interest in the effects of mortality trends on age structure stems from several sources. In many underdeveloped areas spectacular mortality declines since the war have been accompanied by little or no change in fertility. To what extent movements in the death rate tend to raise or lower the relative size of different age classes has an obvious bearing on such questions as the prospective burdens of dependency in these areas, their forseeable rates of saving and investment, and fiscal or other social policies. A second stimulus has come from growing concern over the so-called "aging" of Western populations. According to the traditional view, this development has been the result of long-run downtrends in both mortality and fertility. Recognition in the last few years that the two movements have tended more to offset than to reinforce each other has focused fresh attention on the role of mortality *per se*. Probably a third reason for this interest has been the difficulty of forecasting even the direction of fertility changes in many parts of the world. A frequent expedient in current population projections is to hold fertility constant for "illustrative" purposes, thereby in effect examining mortality changes in isolation. At the same time, the need for more refined evaluation of public health programs and the expansion of social security systems have also activated research along these lines.

The purpose of this paper is to present a general approach to the age mortality problem. In particular, it will be seen that mortality changes tend to have rather limited effect on age

[1] The idea for this paper originated at the 1955 annual conference of the Merrill Center for Economics, where a group of distinguished economists patiently—and vainly—sought enlightenment from the author on the interrelations between vital trends, age composition and economic growth.

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structure. This conclusion has been reached in several studies, but always in connection with specific numerical illustrations only or with special population models.[3] The underlying structure of the probable relations between mortality movements and age in any population has apparently never been investigated.

II

For simplicity all vital events and censuses may be supposed to occur at mid-year, so that the relevant age distributions involve the exact ages 0, 1, . . . (Births and deaths in any census year are assumed to occur "just before" the census.) This is mainly to avoid having to deal with continuous variables; at the same time, no loss of essentials is entailed. A further advantage is that we can make use of exact-age survival rates (the p_x values of the life table), which are more readily available than the corresponding rates for age spans (L_x values). Migration is excluded and constancy of fertility is interpreted in its most usual sense, as unchanging age-specific rates among females. As will be seen, it is convenient to begin by considering females; the corresponding conclusions for males can be readily established later.

[3] *See* Bourgeois-Pichat, J.: Charges de la population active. *Journal de la Société Statistique de Paris,* 91, Nos. 3–4 (March–April, 1950), pp. 94–114; Valaoras, V. G.: Patterns of Aging of Human Populations, in Eastern States Health Education Conference, The Social and Biological Challenge of our Aging Population, New York, Columbia University Press, 1950, pp. 67–85; Lorimer, F.: Dynamics of Age Structure in a Population with Initially High Fertility and Mortality. *Population Bulletin of the United Nations,* (Dec., 1951), No. 1, pp. 31–41; United Nations Population Division: Some Quantitative Aspects of the Aging of Western Populations. *Population Bulletin of the United Nations,* No. 1 (Dec., 1951), pp. 42–51; Dorn, H. F.: Prospects of Further Decline in Mortality Rates. *Human Biology,* December, 1952, 24, No. 4, pp. 235–261; Notestein, F. W.: Some Demographic Aspects of Aging. *Proceedings of the American Philosophical Society,* 98, No. 1 (Feb. 15, 1954), pp. 38–45; Sauvy, A.: Le viellissement des populations et l'allongement de la vie; *Population,* October–December, 1954, 9, No. 4, pp. 675–682; Mortara, G.: Sulla dipendenza della composizione per età di una popolazione dalla mortalità. *Giornale Degli Economisti E Annali di Economia,* January–February, 1955, 14 (New Series), Nos. 1–2, pp. 1–32; United Nations Population Division: The Cause of the Ageing of Populations: Declining Mortality or Declining Fertility? *Population Bulletin of the United Nations,* Dec., 1954, No. 4, pp. 30–38; Coale, A. J.: The Effects of Changes in Mortality and Fertility on Age Composition. Milbank Memorial Fund *Quarterly,* Jan., 1956, 34, No. 1, pp. 79–114.

Parts of the following discussion draw on some suggestions in the last two sources.

Let N_i denote an initial number of persons at age i, $i = 0, 1, \ldots$ Then the population's age distribution is described by the proportions $\frac{N_i}{\Sigma N_i}$. To examine the effects of changing mortality on the distribution at later dates, we first have to define the situation that would have developed under no change. Obviously we can think of values $f_i(t)$, such that $N_i f_i(t)$ would be the number aged i at time t if mortality remained constant after time 0. Letting $c_i(t)$ denote the corresponding age proportions, we have

$$(1) \quad c_i(t) = \frac{N_i f_i(t)}{\Sigma N_i f_i(t)}$$

It will be observed that no restrictions are placed on the demographic background leading up to the initial numbers and none on how age composition would vary over time, given a continuation of the original vital rates.

Suppose instead that survival rates changed "just after" the initial date, and that the new rates continued thereafter. (The effects of successive changes will be considered below.) Then $f_i(t)$ and $c_i(t)$ would be replaced by new values $f_i'(t)$ and $c_i'(t)$, where

$$(2) \quad c_i'(t) = \frac{N_i f_i'(t)}{\Sigma N_i f_i'(t)}.$$

The problem of examining the effects of the given change on age structure may be formulated as follows: How and by what orders of magnitude are $c_i(t)$ and $c_i'(t)$ likely to differ at various times t?[4] More specifically, what sorts of differences can be expected at the pre-adult, adult, and advanced ages, both in the short-run and over longer periods?

[4] Comparisons between $c_i'(t)$ and the original age proportions $\frac{N_i}{\Sigma N_i}$ may be of interest for some purposes but not for the problem at hand. Any actual population would show variations in age structure for some time after all vital rates became fixed. The need for distinguishing between possible comparisons has often been overlooked in the literature, largely because of the tendency to focus on special population types. For example, if the original population were assumed to be stable, $f_i(t)$ would equal e^{rt} for each age i, $\frac{N_i f_i(t)}{\Sigma N_i f_i(t)}$ would equal $\frac{N_i}{\Sigma N_i}$, and the two sets of proportions could be used interchangeably.

The first point to be made is that the measures of mortality change which are relevant for these questions are the proportionate variations in survival rates. Let p_i denote the original rates and p_i' the new ones, and consider any of the initial numbers N_i. The number surviving to time 1 under the original rates would be $N_i p_i$ and under the new rates $N_i p_i'$; hence the ratio of the second to the first group of survivors would be $\frac{p_i'}{p_i}$. The corresponding numbers in the total population would be $B_1 + \Sigma N_i p_i$ and $B_1' + \Sigma N_i p_i'$, where B_1 and B_1' represent the births occurring between times 0 and 1 under the alternative mortality conditions. Since $B'_1 + \Sigma N_i p_i'$ can be rewritten as $\frac{B_1'}{B_1} B_1 + \Sigma N_i \frac{p_i'}{p_i} p_i$, the ratio of this to $B_1 + \Sigma N_i p_i$ is the mean of the ratios $\frac{p_i'}{p_i}$ and $\frac{B_1'}{B_1}$.[5] Moreover, $\frac{B_1'}{B_1}$ is an average of the relative survival changes in the reproductive ages, so that the ratio of total numbers reduces, more simply, to the mean of the various $\frac{p_i'}{p_i}$.[6] Thus, whether the fraction of the population at a given age increased or decreased $[c_i'(1) \gtrless c_i(1)]$, and the extent of the shift, would depend on how the relative survival change at that age compared with the average relative change for the population as a whole.[7]

The relations between $c_i'(t)$ and $c_i(t)$ at any later year

[5] The situation is exactly the same as in deriving the average of n values $v_1, \ldots,$ v_n, with respective frequencies $h_1, \ldots, h_n$. The mean $v = \frac{\Sigma v_i h_i}{\Sigma h_i}$ where the v_i correspond to the above $\frac{p_i'}{p_i}$ and $\frac{B_1'}{B_1}$ and the h_i to $N_i p_i$ or B_1.

[6] Let m_i denote the age-specific fertility rates of persons aged i. Then $B_1 = \Sigma N_{i-1} p_{i-1} m_i$ (where i ranges from about 15 to 49) and $B_1' = \Sigma N_{i-1} p'_{i-1} m_i = \Sigma \frac{p'_{i-1}}{p_{i-1}} N_{i-1} p_{i-1} m_i$. For convenience we shall assume that the female reproductive span begins at 15 and ends at 44, fertility rates at other ages being universally negligible.

[7] In the particular case of the proportions at age 0, $c_0(1)$ and $c'_0(1)$, the corresponding comparison would involve the average change over the reproductive span, rather than for a single age. *See* the preceding footnote.

would be entirely analogous, except that the relative survival changes would be compounded. This is merely a result of the fact that the numbers by age and in the total population would reflect the new survival chances over lengthening intervals of ages. For example, at time 2 the group originally aged i would have either $N_ip_ip_{i+1}$ or $N_ip_i'p'_{i+1}$ survivors under the alternative rates; similarly, the total population would number $B_2 + B_1p_0 + \Sigma N_ip_ip_{i+1}$ or $B_2' + B_1'p_0 + \Sigma N_ip_i'p'_{i+1} = \frac{B_2'}{B_2}B_2 + \frac{B_1'p_0'}{B_1p_0}p_0 + \Sigma\frac{p'_ip'_{i+1}}{p_ip_{i+1}}N_ip_ip_{i+1}$. The ratio $\frac{B_2'}{B_2}$ would be an average of the values of $\frac{p_i'p'_{i+1}}{p_ip_{i+1}}$ in the reproductive ages and $\frac{B_1'p_0'}{B_1p_0}$ would also be a product of two relative survival changes. Accordingly, the ratio of $c_i'(2)$ to $c_i(2)$ at each age could be expressed in terms of such products. At time 3 the relevant variables would be products of three relative changes, at time 4 of four changes, and so on.

It follows that age distribution is unaffected, i.e., $c_i'(t) = c_i(t)$, when the relative changes in survival rates are the same at all ages. Since any average of a group of values, all of which are equal, is the same value, the total population would be changing in the same proportion as the number at each age. This is perhaps so obvious as to require no further explanation, but it is also interesting to see some of the demographic details. Let $p_i' = p_i\ (1 + k)$; that is, suppose the proportionate change k were independent of age. Then at time 1 the survivors of each of the original age groups would be $(1 + k)$ times as numerous as under constant mortality. Since age-specific fertility is assumed to be fixed, the births between times 0 and 1 (hence the "census" number aged 0 at time 1) would also be altered in this proportion, as would the total population. A year later each of the groups aged 2 and over would have had two years of changed survival chances and would be $(1 + k)^2$ as numerous. The number aged 1 would have been altered by a factor of $(1 + k)$ because of the changed number of births a year before, and again by this factor in passing through the first year of life. Finally, the births between times 1 and 2 would have been affected in the same proportion as the numbers in the parental ages. And in general, at each date t the numbers by age and thereby the total population would change by $(1 + k)^t$.[8]

Until now no restrictions have been placed on the nature of the survival changes. A second basic point to note is that the changes which are analytically interesting are increases rather than decreases and of the kinds corresponding to trends rather than to annual variations. Both restrictions have in fact been universally presupposed in the literature, although more often implicitly than explicitly. They will also be assumed here. One reason is that, since sustained survival downtrends have never been documented, investigation of their effects on age composition would have only curiosity value.[9] Another is that the age patterns of change tend to be erratic so long as survival chances are generally falling or, even if rising, when the changes are over very short periods.

In contrast, the longer-run trends on record have typically been of the forms suggested by Table 1. The particular magnitudes are of no concern for the moment, their significant feature being the general way in which they vary with age. It will be observed that the curves of the various columns would resemble a "reversed J" most frequently, a "near U" in some instances and an "L" occasionally. In each case they start from a peak at age 0, drop sharply in the first few years of life and vary mildly from about 5 to between 40 and 60. Thereafter the curves remain fairly horizontal or turn upward, at least to 85. Granted that some of the upper-age measures are of uncertain accuracy, nevertheless an upturn or at least the absence of any marked downturn seems highly probable. Moreover, examination of a large number of changes between successive life tables

[8] To repeat, all changes are relative to the numbers that would be found under constant mortality, not to the original N_i. The equality of $c_i'(t)$ and $c_i(t)$ would hold at each date although $c_i(t)$ would normally be changing.

[9] *See* G. J. Stolnitz, A Century of International Mortality Trends: I. *Population Studies,* July, 1955. 9, No. 1, pp. 24–55.

Table 1. Long-run changes in age-specific survival rates (p_i) for selected countries and periods, females.

Age	Country, Period*																		
	Aust. 1906 to 1947	Bel. 1896 to 1948	Den. 1903 to 1948	Fr. 1901 to 1951	Gy. 1906 to 1933	Neth. 1905 to 1951	N.Z. 1903 to 1951	Nor. 1906 to 1948	Sw. 1906 to 1948	Sz. 1906 to 1951	U.S. 1901 to 1950	Aus. 1903 to 1950	Fin. 1906 to 1951	It. 1906 to 1936	USSR 1897 to 1927	Cey. 1921 to 1946	Ind. 1896 to 1946	Jap. 1923 to 1950	Taiw. 1910 to 1938
0	5.9	10.8	7.7	11.2	12.3	10.9	5.2	4.4	6.0	9.6	9.8	16.2	9.0	6.6	11.6	7.5	11.3	10.1	2.3
1	1.4	3.6	1.6	2.8	3.2	3.3	1.0	1.5	2.0	1.9	3.1	5.4	4.2	3.9	3.6	2.2	.8	3.1	1.2
2	.5	1.8	.6	1.6	1.1	1.4	.4	.7	.9	.8	1.4	3.4	2.2	2.1	2.6	2.4	1.2	1.6	1.6
3	.3	1.1	.4	1.1	.6	.7	.3	.5	.7	.5	.9	1.7	1.7	1.2	1.9	2.8	1.2	1.1	1.5
4	.2	.8	.3	.8	.4	.5	.3	.4	.5	.4	.7	1.1	1.2	.8	1.4	2.2	1.2	.7	1.1
5	.2	.6	—	.6	.3	.4	.2	.4	.5	.3	.5	.8	1.0	.5	1.1	1.7	.9	.4	.8
10	.1	.2	.2	.3	.2	.2	.1	.3	.3	.2	.2	.4	.5	.1	.4	.6	.2	.2	.3
15	.2	—	.2	.4	.2	.3	.2	.4	.4	.3	.3	.4	.5	.3	.2	.4	.6	.7	.3
20	.2	.4	.3	.5	.2	.3	.3	.5	.4	.5	.5	.6	.5	.3	.2	.5	1.0	.7	.7
25	.3	—	.3	.6	.3	.4	.3	.5	.5	.5	.6	.6	.5	.4	.2	.6	.9	.5	.7
30	.4	.4	.4	.6	.3	.4	.4	.5	.5	.5	.6	.7	.5	.4	.3	.7	.6	.5	.8
35	.4	—	.4	.6	.3	.5	.4	.6	.5	.6	.7	.8	.5	.4	.3	.8	.3	.5	.8
40	.4	.5	.4	.6	.4	.5	.4	.6	.5	.6	.7	.8	.6	.4	.4	.8	.3	.5	.7
45	.4	—	.4	.6	.3	.5	.4	.5	.4	.6	.6	.7	.5	.3	.5	.8	.5	.5	.9
50	.3	.5	.4	.6	.3	.6	.4	.5	.4	.7	.7	.9	.6	.3	.7	.8	.7	.5	1.2
55	.4	—	.4	.8	.5	.6	.5	.6	.4	.9	.9	1.2	.6	.3	1.0	1.3	.8	.6	1.4
60	.6	.8	.4	1.1	.7	1.1	.6	.6	.4	1.4	1.1	1.8	.7	.6	1.4	2.0	.6	.7	1.3
65	.9	—	.4	1.7	1.2	1.3	.8	.6	.4	2.3	1.5	2.5	.9	1.0	2.2	2.9	.9	1.0	1.6
70	1.2	1.9	.4	2.5	1.5	2.0	1.2	.7	.5	3.2	2.0	3.4	1.0	1.8	2.6	4.8	2.1	1.7	1.7
75	2.0	—	.5	3.7	2.0	2.5	1.6	.8	.5	4.4	2.6	4.8	1.4	3.0	2.9	9.7	4.9	2.8	1.8
80	1.5	—	−.2	5.4	2.3	3.3	2.0	.9	.4	5.5	3.6	5.5	2.4	4.9	1.6	11.2	10.3	4.7	—
85	.8	—	1.1	5.4	3.0	3.3	.9	.4	.9	6.5	4.5	8.6	2.4	7.3	−2.9	13.4	19.7	8.6	—

Source: Life tables for individual countries.

* Countries are: Australia, Belgium, Denmark, France, Germany, Netherlands, New Zealand, Norway, Sweden, Switzerland, United States, Austria, Finland, Italy, Union of Soviet Socialist Republics (European part), Ceylon, India, Japan, Taiwan. Years are mid-points of periods covered by life tables.

shows that similar overall patterns are found for shorter-run trends.[10]

Thirdly, since the possible increases in survival rates (up to a value of unity) are frequently small relative to their magnitudes, the leeway for actual increases tends to be limited. As Table 1 shows, even long-run changes are likely to be about 1 per cent or less at most ages. It is true that the changes at age 0 are much larger. But it should also be noted that the large majority of the infant survival changes on record have averaged a good deal less than 5 per cent per decade. Furthermore, the possibilities of shifts in age proportions are restricted in another way. Since any rise in the survivors at a given age also involves a rise in total numbers, the two increases tend to offset each other.[11]

The relatively limited effects of most mortality trends on age structure are explained by these two tendencies.

III

Turning to a more detailed analysis, we may think of a (single) survival movement having an age pattern much as in Figure 1. That is, a sharp drop occurs between the ages 0 and 1, followed by far more gradual declines in the next few years. The changes between about 5 and 45 are taken to be perfectly horizontal; as Coale has shown, this assumption generally involves slight errors.[12] Beyond 45 the curve may be supposed to remain relatively horizontal or to rise, the latter being probable. It will be further supposed that we can ignore the ages 90 and over. The proportions over 90 in actual populations are invariably well below 1 per cent and even very large increases in

[10] *See* Stolnitz, *op. cit.*, Part II, to appear in *Population Studies*, 1956. In most instances the life tables were based on the average experience of several-year periods and were about 5 to 15 years apart.

[11] In other words, since any $\frac{c_i'(t)}{c_i(t)}$ consists of a ratio of proportionate survival changes, the effect of a value above unity in its numerator is likely to be dampened because of a value above unity in its denominator.

[12] Coale, *op. cit.*, pp. 97–101; also, Stolnitz, *op. cit.*, Part II.

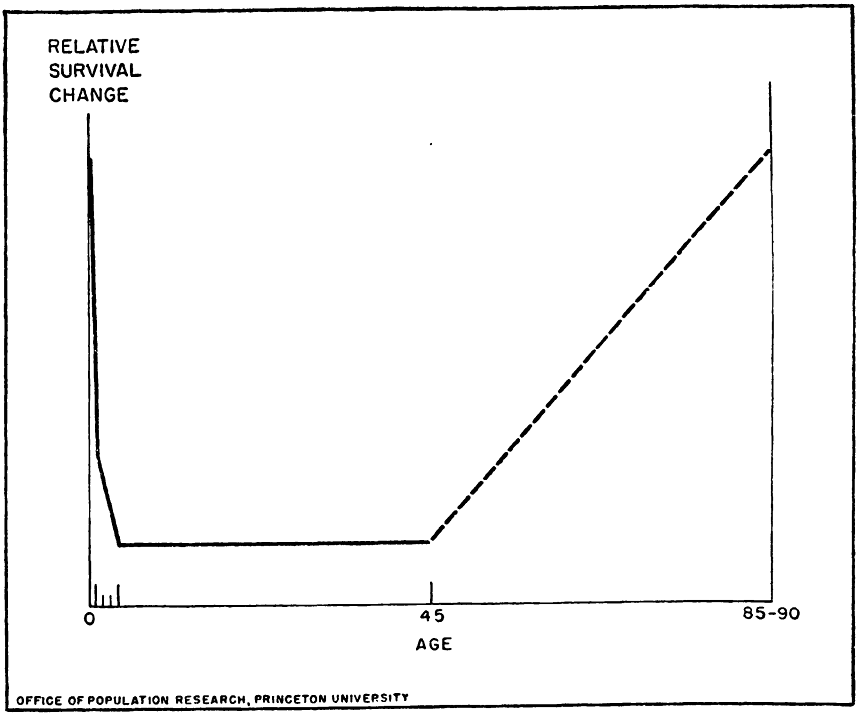

Fig. 1. General pattern of relative changes in age-specific survival rates (p_i).

the future would have negligible effects on the fractions under 90.

The problem of tracing age-structure effects can best be visualized in several stages. To begin with, of course, we postulate a population with given numbers by age as of an initial date and with unchanging vital rates thereafter. The particular numbers and rates are of little or no consequence.

(a) The first step is to consider a population which is initially identical in every respect but suddenly ("just after" time 0) experiences an equal percentage rise in all age-specific survival rates. The rise may be of any magnitude and corresponds to extending the horizontal part of Figure 1 over the entire age scale.

(b) Next, we may think of a second population exactly like the one in (a), except that it has had additional survival gains at the young ages (gains in addition to the horizontal part of

Figure 1). Specifically, such increases will be assumed to be 3 per cent between 0 and 1 $\left(\frac{p_0'}{p_0} = 1.03\right)$ and very nearly 1 per cent in each of the next three years, or up to age 4. Survivors per birth to age 1 in this population would therefore be 3 per cent more numerous than in (a), 4 per cent at age 2, up to 6 per cent at age 4.[13] By assumption a 6 per cent rise would also hold for all later ages. It will be observed that these magnitudes rank among the higher changes encountered in Western populations over periods of 10 to 20 years.[14]

(c) The analysis of the effects of survival trends on age could stop at this point whenever no additional survival increases occur beyond 45. As Table 1 shows, such cases do arise. For purposes of general discussion, however, it is necessary to include a third and last step. This involves a population as in (b) but which has also experienced additional survival increases in the upper ages. If we assume that these increases slope upward at a rate of $\frac{1}{15}$ per cent per year of age from 44 to 88, the additional gains would come to 1 per cent between 58 and 59, 2 per cent between 73 and 74, and 3 per cent between 88 and 89. The total age function of survival changes for this population would therefore resemble a "U," or intermediate in its right-hand portion between the "reversed J" and "J" functions found in Table 1.

Reduced to essentials, the problem is to compare the age distribution that would develop in (b) with that in (a) and then

[13] These above numbers were chosen partly in the light of empirical considerations and partly to avoid fractions as much as possible. Strictly, a rise of 3 per cent in p_0, followed by a rise of 1 per cent in p_1 through p_3, means that the number reaching age 2 would increase by a factor of (1.03) (1.01), the number reaching age 3 by (1.03) $(1.01)^2$, up to (1.03) $(1.01)^3$ at age 4. The differences between these values and the ones in the text are negligible.

The technique of adding percentage changes rather than compounding ratios will be used throughout. Although it involves larger errors than the foregoing at some points, the biases are generally negligible. The main exception is at the older ages, where more extensive compounding is involved, this difficulty can be readily resolved, however, by supposing that the upper-age survival movements are in fact slightly smaller than the ones cited.

[14] *See* Table 1. The effects of much larger changes, such as have occurred in many "under-developed" areas since the war, are illustrated in Part IV.

to allow for the further effects of the upper-age changes in (c). The use of specific numbers at this point is largely for expository convenience. Alternative assumptions could be handled in analogous fashion and would give rise to analogous age-structure effects. Indeed, as will be discussed later, the effects resulting from one set of survival changes will often provide quick estimates of the effects in other situations.

(a) *Equal Relative Survival Increases at All Ages.* Since such changes would lead to the same age distribution as would exist with no change, this population may be substituted as the standard for further comparisons. The advantage of so doing is that we have to trace the changes in numbers at various individual ages and it is convenient to ignore the increases which occur in equal proportions everywhere. If persons aged 15–29 are 1 per cent more numerous because of a uniform gain in survival rates and an additional 5 per cent because of larger young-age gains, the latter value alone will affect the size of the group relative to the total population.[15]

Accordingly, all further references to changing numbers are in relation to the population under (a), not to the original population.

(b) *Additional Increases in Young-Age Survival Rates.* Given the assumed numerical changes, the situation at time 1 would be as follows: The number aged 0 would be the same as in (a), the number at age 1 would be 3 per cent higher and those at each year from 2 to 4 would be 1 per cent higher.[16] Since none of the groups 5 and over would have been affected as yet, the rise in the total population would be very small (far below $\frac{1}{2}$ per cent as a rule). The percentage increases in the age proportions below 5 would therefore be very nearly the same as the gains in population numbers, while the proportions

[15] Alternatively, of course, we could simply have assumed that the changes between 5 and 45 were zero.

[16] The lack of any change at 0 under our simplifying assumptions corresponds to the fact that the number 0–1 in an actual population would only partly reflect a previous rise in the infant survival rate.

at each later age would fall to the extent that total numbers had risen.

The corresponding calculation for later periods could be carried out in as much detail as seemed desirable. For seeing general patterns, however, we may limit the discussion to 15-year intervals of age and time. In this way we can most conveniently trace the changing numbers in the reproductive ages (15–44).

Time 15: The only 15-year groups with increased numbers at this date would be those 0–14 and 15–29. Since the sub-group 15–19 would be 2–3 per cent more numerous, the gain for the toal group 15–29 would be very close to 1 per cent.[17] Among those under 15, persons in each of the ages 5–14 would be 6 per cent more numerous. The rise in the 0–4 sub-group would come almost entirely from additional survivors per birth and amount to about 4 per cent. (The increases in survivors per birth by single years of age would be 0, 3, 4, 5 and 6 per cent, respectively, or an average of 3.6 per cent. To a slight extent, this sub-group would be further augmented by the rise in births shortly before time 15 from persons 15–19.) For practical purposes, therefore, we can assume a 5 per cent increase for the group as a whole.

Time 30: Persons aged 30–44 at this date would be 1 per cent more numerous, the same rise as was found for this cohort 15 years earlier. However, the cohort aged 15–29 would be 6 per cent larger (against the previous 5 per cent), since each annual sub-group would have had time to benefit from all the young-age survival gains.

The group aged 0–14 would be augmented in two ways. First, there would be about 5 per cent more survivors per birth, as at time 15. And secondly, the births themselves would be more numerous, since the reproductive cohorts would have gained in size. "Just before" time 30 the rise in births to persons 15–29 would be 6 per cent, this being the rise in their number at each

[17] The increases by single years of age from 15 to 19 would be 6, 3, 2, 1 and 0 per cent, respectively, and the gain for the entire sub-group would tend to be somewhat above the simple average of these values. Similarly, the rise in the total 15–29 group would slightly exceed the average of the increases in its 5-year components (3, 0, 0).

year of age. At time 15 the group 15–29 was 1 per cent larger, so that the average increase for the period as a whole can be taken as some 3.5 per cent. To a close order of approximation this would also be the rise in births from persons at these ages, again for the period as a whole.[18] In addition, births to persons above 30 would also have increased, say by $\frac{1}{2}$ of 1 per cent, as a result of the augmented numbers passing from 25–29 at time 25 to 30–34 at time 30.

In order to combine these component increases, we can use the fact that women 15–29 rarely contribute less than half or more than three-fourths of total births. For present purposes we can assume a 60–40 distribution, hence that the period rise in births would be about 2.5 per cent.[19]

In combination with the 5 per cent gain in survivors per birth this would raise the 0–14 group by 7.5 per cent. (By time 34, when all members of this group were aged 4 and over, the rise would go up to 8.5 per cent.)

Time 45: At this date persons 30–44 would be 6 per cent more numerous, compared to the 1 per cent increase found for these ages at time 30. Thus the average rise over the intervening period would be 3.5 per cent. The corresponding increases of persons 15–29 would be 8.5 and 6 per cent, or an average of somewhat above 7 per cent. Accordingly, the gain in total births between time 30 and time 45 would be very nearly 6 per cent ($.6 \times 7.0 + .4 \times 3.5$) and the number 0–14 would be more

[18] Since fertility rates typically increase from 15 to about 25, the gain in births could be expected to rise from less than 1 per cent at time 15 to the final 6 per cent. Thus the cited value is probably somewhat of an over-estimate.

[19] Let B denote the births that would have occurred with no young-age survival increases, and b and b′ the corresponding births to women 15–29 and 30–44. Then the new number would be $1.035b + 1.005b'$, the absolute rise $.035b + .005b'$ and the proportionate change $.035\,\frac{b}{B} + .005\,\frac{b'}{B}$. The cited overall gain comes from setting $\frac{b}{B}$ equal to .6 and $\frac{b'}{B}$ to .4. Although the fractions used in a concrete problem should be adapted to the given population, practically any reasonable choice would suffice. In the present case, for example, setting $\frac{b}{B}$ equal to .50 or .75 would imply a gain in total births of 2.0 or 2.8 per cent and subsequent gains would soon converge to a steady increase of about 3 per cent per period, as discussed in the text.

numerous by 11 per cent. The group 45–59 would be larger by 1 per cent and all older cohorts would still be as in (a).

Later Dates: Beginning with time 45 the gains in births would climb at a steady rate of close to 3 per cent per period, from the 6 per cent just noted to 9 per cent between time 45 and 60, and so on.[20] Table 2 presents the underlying computations. Thus at time 60 the increase in the number 30–44 would be 8.5 per cent, compared to 6 per cent at time 45, and the resulting rise in births about 7 per cent. The corresponding end-date increases for 15–29 would be 12 and 8.5 per cent, respectively, and the rise in births nearly 10 per cent. The overall gain in births would therefore be 9 per cent and the survivors aged 0–14 would be augmented by 14 per cent (compared to the 11 per cent rise for this age span at time 45).

And in general, the increases in numbers 15–29 and 30–44, hence in births, would continue to rise by very nearly 3 per cent in each later 15-year period. In turn, since the new survival rates hold for all cohorts of persons born after time 0, the same

Table 2. Computation of per cent increases in births during successive 15-year periods, under 6 per cent rise in 0–4 survivorship.[1]

Time	Per Cent Rise in Number[a]		Average Per Cent Rise, Previous 15 Years[b]		Per Cent Rise in Total Births, Previous 15 Years[c]
	15–29	30–44	15–29	30–44	
(1)	(2)	(3)	(4)	(5)	(6)
15	1.0	0.0	0.0	0.0	0.0
30	6.0	1.0	3.5	0.5	2.5
45	8.5	6.0	7.0	3.5	6.0
60	12.0	8.5	10.0	7.0	9.0
75	15.0	12.0	13.5	10.5	12.0
90	18.0	15.0	16.5	13.5	15.0

[1] *See* text and footnotes 16–19, 27.

[a] Rise at time 30 and each later date equals 6 per cent plus rise shown in column (6) of preceding row. Rise in number 15–29 at time 15 is approximately one third of rise in number 15–19.

[b] Average of rise shown in same and preceding rows of column (2) or column (3). Entries used as estimates of increases in births from cited ages.

[c] Equal to .6 × column (4) + .4 × column (5).

[20] This merely illustrates the general theorem discussed in Part IV: Given a total rise of x per cent in young-age survivorship, then the gains in births will approach a steady rate of increase of $\frac{x}{2}$ per cent per period.

relative gains would come to be repeated between the cohorts reaching a given age span at successive "census" dates and between successive age cohorts at the same date. Thus at time 90, when the population of time 0 would have disappeared, the gains would be 6 per cent for persons 75–89, 8.5 per cent for those 60–74, 12 per cent for 45–59, back to 18 per cent for 15–29 and 20 per cent for 0–14 (which would shortly become 21 per cent for this cohort). At time 105 the corresponding gains in numbers by age would be 8.5, 12, 15, back to 21 and 23 per cent; at time 120 they would be 12, 15, 18, back to 24 and 26 per cent; and so on.

It follows that essentially no further effects on age structure would occur after time 90. Each subsequent 15-year period would see an additional gain of 3 per cent (or very nearly this amount) at all age groups and in the total population, compared to the numbers that would have been found under unchanging mortality. Since the latter numbers would themselves be changing by a very nearly constant rate per period, the ratio of the two sets of age proportions at time 90 would tend to be repeated at all later "censuses."[21] For example, consider the dates 90 and 105. Let k denote the stable-age rate of growth per period implied by the original mortality situation. For each age we have $N_i f_i(105) = kN_i f_i(90)$, to a close order of approximation; similarity, $\Sigma N_i f_i(105) = k\, \Sigma N_i f_i(90)$. Let r_i denote the relative increases in the various numbers by age at time 90, or $\frac{N_i f_i'(90)}{N_i f_i(90)}$, under the given survival gains. Then the ratio of total numbers $\frac{\Sigma N_i f_i'(90)}{\Sigma N_i f_i(90)}$ would equal $\frac{\Sigma r_i N_i f_i(90)}{\Sigma N_i f_i(90)}$ and the ratio of the two age proportions at any age i would be r_i divided by this value. At time 105 the relative increase in the number at i, $\frac{N_i f_i'(105)}{N_i f_i(105)}$, would be $1.03 r_i$, as we have just seen. The rela-

[21] A population subject to constant age-specific fertility and mortality will assume a fixed, "stable-age" distribution within a few generations. *See* Lotka, A. J.: *Théorie analytique des associations biologiques,* Part XII, No. 780 in the series ACTUALITÉS SCIENTIFIQUES ET INDUSTRIELLES. Paris, Hermann et Cie., 1939, especially pp. 64–77.

tive increase in total numbers would also have risen by the factor 1.03 since

$$\frac{\Sigma N_i f_i'(105)}{\Sigma N_i f_i(105)} = \frac{\Sigma 1.03\, r_i N_i f_i(105)}{\Sigma N_i f_i(105)} = \frac{1.03\, \Sigma r_i k N_i f_i(90)}{\Sigma k N_i f_i(90)}.$$

Accordingly, the shift in age proportions at time 90 would again be found at time 105 and, by the same argument, at all later dates.

For this reason Table 3, which shows the increases in members by age at successive dates, stops at time 90.

The remaining problem, therefore, is to see how total population size would change at times 15, 30, . . . , 90, in order to convert the above changes in numbers to changes in proportions. Two points should be noted in this connection. Obviously, the conversion factor at any date would be the same for all ages. Secondly, given the increases in numbers by age, the resulting total rise would depend on the distribution that would have been found under no change. Thus consider time 15, when the main variation was a rise of 5 per cent in the group 0–14. Suppose this group would otherwise have included 40 per cent of the population, a common magnitude in high-fertility areas. Then the rise in total numbers would be very nearly 2 per cent and the change in the proportion under 15 about 3 per cent $\left(\frac{1.05}{1.02} \times 100\right)$. On the other hand, suppose that the proportion

Table 3. Per cent increases in numbers by age at successive 15-year "censuses," under 6 per cent rise in 0–4 survivorship.[1]

Time	Age Group					
	0–14	15–29	30–44	45–59	60–74	75–89
(1)	(2)	(3)	(4)	(5)	(6)	(7)
15	5.0	1.0	0.0	0.0	0.0	0.0
30	7.5	6.0	1.0	0.0	0.0	0.0
45	11.0	8.5	6.0	1.0	0.0	0.0
60	14.0	12.0	8.5	6.0	1.0	0.0
75	17.0	15.0	12.0	8.5	6.0	1.0
90	20.0	18.0	15.0	12.0	8.5	6.0

[1] *See* text for derivation.

would have been as low as 20 per cent, or not far from current levels in some Western areas. In this case the rise in total numbers would be only 1 per cent and the change in the proportion under 15 about 4 per cent. The group 15–29, which had risen by 1 per cent, would show a – 1 or a 0 per cent change in relative size and the proportions for all older groups would fall by 2 or 1 per cent.

Since no restrictions are placed on the age distribution serving as standard, it is useful to see how total numbers might change in a wide variety of situations. Fortunately, this question poses few difficulties. As can be seen from Table 4, the results obtained by applying the preceding increases by age to the "old" population of France in 1946, and the "young" population of Brazil in 1940, come close to encompassing the range of demographically interesting possibilities.[22] It will also be noted that the increases based on the Brazilian distribution

Table 4. Per cent increases in total numbers at successive 15-year "censuses," under 6 per cent rise in 0–4 survivorship and illustrative age distributions.[1]

Time	Age Distribution from			
	France, 1946	Brazil, 1940	Sample of Censuses[2]	
			Low	High
(1)	(2)	(3)	(4)	(5)
15	1.3	2.5	1.3	2.7
30	3.1	5.2	3.1	5.4
45	5.6	8.2	5.6	8.4
60	8.6	11.3	8.6	11.5
75	11.7	14.4	11.7	14.6
90	14.8	17.4	14.8	17.6

[1] Increases by age are from Table 3. For each age distribution, the increase in total number at any date was obtained by multiplying the appropriate increases by age by the corresponding age proportions and then adding the products. Proportions reported for all persons 75 and over were used for 75–89 and persons of unknown age were excluded. *See also* footnote 21.

[2] To time 45 based on 142 age distributions, thereafter on 129 distributions (including France and Brazil) in United Nations, Demographic Yearbook, 1948–1954, and on selected Swedish censuses, 1750–1870.

[22] Use of the same age distribution as a standard for successive dates does not, of course, conform to our model, which is more general. Strictly, the French and Brazilian distributions should have been projected on the basis of the vital rates existing in 1946 and 1940, respectively. The contrasts between the two distributions are sufficiently great, however, to suggest the range of increase in total population for each date separately.

always exceed the ones deriving from the French data. This follows from our previous point that any change in total numbers is a weighted average of the changes by age. An age distribution such as Brazil's would give greater weight to the largest increases in numbers, which are always at the younger ages, and lesser weight to the increases at the older ages. In terms of regional comparisons, populations in Latin America, Africa and Asia would tend to have the largest increases in total numbers and Western countries the smallest, with intermediate changes in Eastern and Southern Europe. Whatever the population type, however, the increases in total numbers would tend to fall into a regular pattern of variation over time, the added gain per period approaching the 3 per cent increment in births.

The age-distribution effects shown in Table 5 are therefore suggestive in several respects. (All entries derive directly from Tables 3 and 4; for example, the increase of 3.7 per cent in the

Table 5. Per cent changes in age proportions at successive 15-year "censuses," under 6 per cent rise in 0–4 survivorship and two contrasting age distributions.[1]

Time	Age Group					
	0–14	15–29	30–44	45–59	60–74	75–89
(1)	(2)	(3)	(4)	(5)	(6)	(7)
	1. Age distribution in France, 1946					
15	3.7	−.3	−1.3	−1.3	−1.3	−1.3
30	4.3	2.8	−2.0	−3.0	−3.0	−3.0
45	5.1	2.7	.4	−5.3	−5.3	−5.3
60	5.0	3.1	−.1	−2.4	−7.0	−7.9
75	4.7	3.0	.3	−2.9	−5.1	−9.6
90	4.5	2.8	.2	−2.4	−5.5	−7.7
	2. Age distribution in Brazil, 1940					
15	2.4	−1.5	−2.4	−2.4	−2.4	−2.4
30	2.2	.8	−4.0	−4.9	−4.9	−4.9
45	2.6	.3	−2.0	−6.7	−7.6	−7.6
60	2.4	.6	−2.5	−4.8	−9.3	−10.2
75	2.3	.5	−2.1	−5.2	−7.3	−11.7
90	2.2	.5	−2.0	−4.6	−7.6	−9.7

[1] Based on Tables 3 and 4. *See* text.

first row is equal to $\left[\frac{1.050}{1.013} - 1\right] \times 100$.) For reasons just noted, they represent very nearly the limiting shifts to be expected from the assumed survival changes. The increases in the French age proportions would in every case be larger, or the decreases smaller, than the corresponding entries for Brazil. Analogous contrasts would be highly probable, had we selected any other young-age survival changes or practically any other areas of similar backgrounds. At the same time, the differences between the two parts of the table are perhaps less important than the secondary effects suggested by both. An increase of 5 per cent in the proportion 0–14 would raise the 1946 French value from 20.6 per cent to only 21.6 and the 1940 Brazilian value from 42.0 to 43.0. It is true that most of the percentage shifts in the proportions over 60 are larger. But it should be recalled that no allowance has been made for additional upper-age survival gains. Moreover, even if such gains did not occur, the absolute variations in proportions would again be small. A decline of 10 per cent in the proportion over 75 would lower the 1946 French fraction from 4.0 to 3.6 per cent and 1940 Brazilian measure from .9 to .8 per cent. Such variations are well within the range of values encountered in empirical studies.[23]

Finally, mortality declines up to 45 have a decided tendency to make a population younger. This will not only be the case when the upper-age survival gains remain at the 5–45 level, as here, but also when they rise to only a small fraction (approximately less than one third) the total 0–4 gain. The last has in fact been the usual situation in Western countries over most of the last century.

(c) *Additional Increases in Upper-Age Survival Rates.* The problem of modifying the foregoing for survival changes beginning about age 45 is relatively simple in one respect. Since such changes come at practically the end of the reproductive span, they have very little influence on numbers of births. The

[23] *See* for example, Sauvy, *op. cit.*, p. 679. Or, to put the matter another way, the small absolute differences in proportions often cited may in fact represent fairly substantial changes in percentage terms.

increases in numbers under 45 from a young-age survival movement can therefore be assumed to remain unaltered by developments at later ages. Only the age proportions under 45 are affected and all in equal degree. For example, suppose that upper-age survival gains raised total population at a given date by an additional 1 per cent, compared to what it would be in (b). Then the proportions 0–14, 15–29 and 30–44 would all be lowered by the same margin.

In another particular, however, the analysis is more complicated. The full effect of a young-age survival gain on the size of a given birth cohort is manifested within a short interval of time. Thus the entire (percentage) rise for each 15-year cohort in (b) was experienced within five years of its period of birth, whether the rise came from added survivors per birth or from added births. In contrast, the increased numbers resulting from upper-age gains tend to be cumulative. Under the gains assumed here, for example, an annual group reaching age 45 would rise by an additional $\frac{1}{15}$ per cent (over whatever increase had occurred at earlier ages). A year later, the same cohort would gain another $\frac{2}{15}$ per cent or a total rise of $\frac{3}{15}$ per cent. Two years later the total gain would be $\frac{6}{15}$ per cent and so on to age 89, when the total rise would be 69 per cent.[24]

For this reason, and to take account of both transitional and long-run effects, it is again necessary to deal with individual dates. This time, however, the increases in numbers are to be understood as being in addition to the ones already cited in (b).

Time 15: Consider the group 45–59. The sub-group just reaching age 45 would be more numerous by only $\frac{1}{15}$ per cent;

[24] The last figure comes from the sum $\frac{1}{15}+\frac{2}{15}+\ldots+\frac{45}{15}$ or $\frac{1}{15}\times\frac{45\times 46}{2}$, following the familiar equality $1+2+\ldots+n=\frac{n(n+1)}{2}$.

the one aged 46 would have gained $\frac{1}{15}+\frac{2}{15}$ per cent; up to the one aged 59, whose gain would be 8 per cent $\left(\frac{1}{15}\times\frac{15\times 16}{2}\right)$. The gain for the group as a whole would be a weighted average of these individual gains, the weights being the proportions at the various ages if there were no upper-age survival gains. Examination of a variety of censuses shows that the weights can be assumed to decline linearly and that setting the weights at about .1 for age 45 down to about .03 for age 59 is likely to entail only minor errors. Thus the gain for the total group would be very nearly 2.2 per cent, the figure used in Table 6.[25]

Among those 60–74, the oldest annual sub-group would have gained $1\frac{1}{15}$ per cent between 59 and 60, another $1\frac{2}{15}$ per cent at 61, up to $1\frac{15}{15}$ per cent at 74, or a total gain of 23 per cent $\left(15+\frac{1}{15}\times\frac{15\times 16}{2}\right)$. The second oldest sub-group moved through the

Table 6. Per cent increases in numbers by age at successive 15-year "censuses," under 6 per cent rise in 0–4 survivorship and linear rise in old-age survivorship up to 3 per cent.[1]

Time	Age Group					
	0–14	15–29	30–44	45–59	60–74	75–89
(1)	(2)	(3)	(4)	(5)	(6)	(7)
15	5.0	1.0	0.0	2.2	14.6	29.6
30	7.5	6.0	1.0	2.2	16.8	44.1
45	11.0	8.5	6.0	3.2	16.8	46.3
60	14.0	12.0	8.5	8.2	17.8	46.3
75	17.0	15.0	12.0	10.7	22.8	47.3
90	20.0	18.0	15.0	14.2	25.3	52.3

[1] Columns (2)–(4) as in Table 3. *See* text for derivation of columns (5)–(7).

[25] With a substantially slower decline in weights, from about .08 to .04, the gain would be 2.6 per cent; with a steeper decline, from .12 to .01, it would be 1.7 per cent. For the above estimate to be in appreciable error the weights at the upper half of the age span would have to be consistently larger than the weights at the lower half.

ages (58–73) yielding increases of 1 per cent through $1\frac{14}{15}$ per cent; the third oldest $\frac{14}{15}$ through $1\frac{13}{15}$ per cent; down to the youngest sub-group, with $\frac{2}{15}$ through $1\frac{1}{15}$ per cent. It will be noted that any sub-group would gain $\frac{1}{15}$ per cent more than the next younger sub-group during each calendar year, hence would experience a 1 per cent larger gain for the entire 15-year period. The total gains by sub-groups would therefore range from 23 down to 9 per cent and the weighted average would come to about 14.6 per cent. Finally, since the same comparative gains would hold for successive sub-groups under and over 75, the gains for persons 75–89 would range from 24 up to 38 per cent, and the overall gain would be about 29.6 per cent.[26]

Time 30: Once again the group 45–59 would have sub-group increases ranging from $\frac{1}{15}$ to 8 per cent and an overall rise of 2.2 per cent. Persons aged 60–74 would have been 30–44 at time 0, hence would also have experienced all the survival gains to their attained age. By age 59 the total increase for each annual sub-group would be 8 per cent, and the further increases would be $1\frac{1}{15}$ per cent for those just reaching 60 at time 30, $1\frac{1}{15} + 1\frac{2}{15}$ for those 61, up to 23 per cent $\left(15 + \frac{1}{15} \times \frac{15 \times 16}{2}\right)$ for those aged 74. Compared to the population having no additional upper-age survival gains, therefore, the increases for these successive sub-groups would vary from $9\frac{1}{15}$ up to 31 per cent, with an estimated rise of 16.8 per cent for the group as a whole.

The youngest of the sub-groups aged 75–89 was 45 at time 0

[26] Using the alternative weights of the preceding footnote, the increases for the 60–74 and 75–89 groups would be 13.7 or 15.2, and 28.7 or 30.2, respectively. The range of variation would be somewhat larger at later dates, but again by unimportant margins.

and would have gained $\frac{2}{15} + \ldots + \frac{15}{15} + 1\frac{1}{15} + \ldots + 1\frac{15}{15} + 2\frac{1}{15}$ per cent or a total of 33 per cent (the sum $\frac{1}{15} + \ldots + \frac{15}{15} = 8$ appears twice, the value 1 appears fifteen times and the value 2 once). The next older sub-group (aged 46 at time 0) would have the same gains, except that the first value of $\frac{2}{15}$ would be replaced by $2\frac{2}{15}$ in passing from 75 to 76; its total gain would therefore be 2 per cent larger, or 35. The gains for successive pairs of older sub-groups would rise similarly, from 37 to 61 per cent, and the increase over (b) for the full group would be about 44.1 per cent.

Time 45: The computations just carried out would now apply to the groups 45–59 and 60–74. The only new situation would involve the 75–89 cohort, whose members were 30–44 at time 0. Each sub-group would have had all the upper-age survival gains to 74, or a total of 31 per cent, and the further gains would range from $2\frac{1}{15}$ per cent for those aged 75 to 38 per cent for those 89. The weighted average could be expected to fall in the neighborhood of 46.3 per cent.

Later Dates: Time 45 marks the end of all transitional effects. From this point on the percentage gain over (b) at any age from 45 to 89 would be the same at all dates. Whatever their gains before 45, the groups aged 45–59, 60–74 and 75–89 at any given date would always be augmented by an additional 2.2, 16.8 and 46.3 per cent, respectively.

Combining the effects of younger-age and upper-age gains would not, of course, affect the attainment of a stable-age situation by time 90. Any later "census" would show increases in numbers by age which would be 3 per cent above the increases of the preceding "census" (15 years earlier). As in (b), the cohorts found at successive dates in any given age interval would have the same (new) survival rates from birth to time of enumeration and hence the same percentage increase in survivors per birth. Accordingly, the difference between their increases would be entirely determined at birth.[27]

Tables 6–8 show the effects on numbers by age, total numbers and age proportions, respectively, when both young-age and upper-age survival changes are considered. The first consists simply of the increases in Table 3 plus the upper-age increases just cited. Similarly, the gains in total numbers of Table 7 come from adding the gains in Table 4 (for France and Brazil) to the ones that would result from applying these upper-age increases to the same underlying distributions. It will be seen that the effect of assuming a U-shaped curve of survival changes is to reverse a previous comparison; here the French distribution leads to the larger increases in total numbers. In another respect, however, the patterns in Tables 4 and 7 are similar, with the increments per period in each soon converging to a 3 per cent level.

Table 8 is based on Tables 6 and 7. As in Table 5, the most appreciable shifts in individual age proportions are at the old ages. This time, however, the latter show marked increases, rather than decreases, in relative size. As a result, there is a

Table 7. Per cent increases in total numbers at successive 15-year "censuses," under 6 per cent rise in 0–4 survivorship, linear rise in old-age survivorship up to 3 per cent, and two contrasting age distributions.[1]

TIME	AGE DISTRIBUTION IN	
	France, 1946	Brazil, 1940
(1)	(2)	(3)
15	4.9	3.3
30	7.5	6.2
45	10.1	9.3
60	13.1	12.5
75	16.3	15.6
90	19.4	18.6

[1] *See* Table 4, first footnote.

[27] The one contrast with (b) is that 3 per cent would no longer represent the difference between the increases of successive pairs of age groups as of the same date. For example, the increase in the number of persons 30–44 would be less than 3 per cent above the increase among those 45–59, since the latter would have experienced additional survival gains.

pronounced tendency toward a decline in the relative numbers at the main working ages (15–60) and toward an increase in the numbers at the ages of partial or total dependency. This combination of shifts can be expected whenever the upper-age gains rise to levels comparable to those at the opposite end of the age scale.

It is also worth illustrating the effects on age that would have been found, had we assumed the more common pattern of a "reversed J." Thus, with a linear rise of $\frac{1}{30}$ per cent in upper-age survival gains (up to a $1\frac{1}{2}$ per cent rise at 88–89), the increases in numbers by age beyond 45 would be half the ones cited in (c); similarly, the additional increases in total numbers would be half the differences between Tables 4 and 7.

Table 8. Per cent changes in age proportions at successive 15-year "censuses," under 6 per cent rise in 0–4 survivorship, linear rise in old-age survivorship up to 3 per cent, and two contrasting age distributions.[1]

Time	Age Group					
	0–14	15–29	30–44	45–59	60–74	75–89
(1)	(2)	(3)	(4)	(5)	(6)	(7)
	1. Age distribution in France, 1946					
15	0.1	−3.7	−4.7	−2.6	9.2	23.5
30	0.0	−1.4	−6.0	−4.9	8.7	34.0
45	0.8	−1.5	−3.7	−6.3	6.1	32.9
60	0.8	−1.0	−4.1	−4.3	4.2	29.4
75	0.6	−1.1	−3.7	−4.8	5.6	26.7
90	0.5	−1.2	−3.7	−4.4	4.9	27.6
	2. Age distribution in Brazil, 1940					
15	1.6	−2.2	−3.2	−1.1	10.9	25.5
30	1.2	−0.2	−4.9	−3.8	10.0	35.7
45	1.6	−0.7	−3.0	−5.6	6.9	33.9
60	1.3	−0.4	−3.6	−3.8	4.7	30.0
75	1.2	−0.5	−3.1	−4.2	6.2	27.4
90	1.2	−0.5	−3.0	−3.7	5.6	28.4

[1] Based on Tables 6 and 7. *See* text.

Some of the resulting changes in age proportions would be as follows:

Time	*0–14*	*15–29*	*30–44*	*45–59*	*60–74*	*75–89*
	1. Based on Age Distribution in France, 1946					
15	1.8	–2.0	–3.0	–1.9	4.1	11.3
45	3.0	0.6	–1.7	–5.3	0.6	14.3
75	2.6	0.9	–1.8	–3.9	0.4	8.9
	2. Based on Age Distribution in Brazil, 1940					
15	2.0	–1.8	–2.8	–1.7	4.3	11.6
45	2.0	–0.3	–2.6	–6.2	–0.4	13.2
75	1.7	0.0	–2.6	–4.7	–0.5	8.0

A tendency to declining relative size of labor force is again indicated, although to a much lesser extent than in Table 8. This last is, of course, to be expected, since smaller old-age survival gains are assumed. A more important result is that nearly all of the age-distribution shifts turn out to be secondary, although the underlying survival movements are by no means minor. This is generally true even if we measure the shifts in percentage terms. In absolute terms, only one of the age proportions in France and Brazil would shift by as much as a single percentage point.

IV

We are now in position to relax or at least reconsider some of our original assumptions.

1. One that is readily disposed of, since there is so little to say, is that survival changes occur instantaneously. As already indicated, year-to-year changes tend to vary irregularly with age, hence are not likely to yield conclusions of general interest. And even if this were not the case, the effects on age structure could only be brought out by means of extremely detailed computations. The age-mortality problem necessarily involves step-by-step projections, unless we are willing to limit ourselves to very special types of populations and time paths of change. Finally, in so far as the irregularities of annual changes cancel

each other over longer periods, the tendencies would be analogous to the ones already described. For example, suppose the specific survival changes assumed above had developed over a 15-year period, fertility rates again remaining constant and mortality first becoming fixed by time 15. Then the numerical effects on age structure would be as before except in one or two minor particulars. Since the rise in the numbers 0–14, 15–29 and 45+ at time 15 would be far below the ones shown in Table 6, we could proceed very nearly as if no changes at all had occurred before this date $\left[\frac{c_i'(15)}{c_i(15)} \approx 1 \text{ for all } i\right]$. At time 30, therefore, the increases by age and in total numbers would be almost the same as indicated previously for time 15. In turn, the corresponding lag would hold for each later "census" and very much the same stable-age situation as before would develop by time 105.

2. Secondly, the increases in Tables 3 and 4 under the given young-age survival movements can be readily adapted to other numerical assumptions (again using 15-year intervals of time and age as units). The central variable in any situation is the total percentage rise (over the 5–45 level) in the chances of surviving from birth to about age 5, i.e., the sum of the rises in the initial 4 to 5 years of life. The percentage increase in the size of each cohort born after the survival movement can be regarded as the outcome of an increase in survivors per birth and of an increase in births. The first of these components is (or in the case of the 0–14 group soon becomes) exactly equal to the total 0–5 rise. The second component comes close to being proportional to the same rise, the proportion depending on the cohort's period of birth. If the births are for the period 0–15, their increase is very small; if for the period 15–30, their increase is not far from half the 0–5 change; for 30–45 the increase almost equals the change; for 45–60 the increase is 1.5 times the change, and so on.[28] Accordingly, given two 0–5

[28] As the first of the cohorts experiencing the whole 0–5 survival rise fills out the reproductive ages, its increase is averaged with the (somewhat above) zero increase

(Continued on page 205)

survival changes, X and Y, the percentage increases in numbers by age at a given date under Y would be $\frac{Y}{X}$ times the corresponding increases under X, to a close order of approximation. The total population changes would be modified in the same way. In particular, given any assumed Y, the limiting effects on age can be approximated by applying the factor $\frac{Y}{6}$ to Tables 3 and 4 and then deriving the changes in age proportions as in Table 5.

The effects of upper-age survival changes are less amenable to rapid estimation, partly because more ages are involved and partly because the patterns of change may be more variable. To the extent that the curve of such changes can be regarded as linear, of slope S, the factor $S \div \frac{1}{15}$ can be applied directly to the preceding data (i.e., to the differences between Tables 3 and 6 and between Tables 4 and 7).[29] In fact, linearity from about 45 to only 75 would be sufficient for many purposes. Use of the 45–75 slope would yield quick approximations of the changing numbers at these ages and also in the total population; only the 75–89 estimates would be in risk of substantial error.[30]

Many of these considerations are illustrated in Table 9, which presupposes a 12 per cent rise in 0–4 survivorship and consis-

of the next older cohort, in determining the average growth at such ages over the preceding periods. The lag between the growth at the younger and older reproductive ages complicates the process somewhat, but essentially the tendency is for the first addition in total number of births to approach half the gain in survivors per birth. This addition tends to raise the size of the next following parental cohort, hence the births of the next following period, by the same margin. Once started (about time 45), the process of very nearly constant further additions per period continues indefinitely. *See* Table 2.

[29] In some instances, of course, the distribution within broad age groups should also be taken into account. *See* footnotes 24 and 25.

[30] Other possibilities are that the upper-age changes are decidedly not linear or first become manifest well beyond 45. To assume linearity beyond 45 in such cases would probably not lead to serious errors in the age-proportion shifts found below 45, since the errors in estimated total numbers would tend to be limited. Thus the pattern of effects would still be similar in overall terms to those already discussed. On the other hand, the errors in the proportions 45 and over might be large enough to require a completely new set of estimates.

tently rising upper-age gains, from $\frac{2}{15}$ per cent at 44–45 to 6 per cent at 88–89. All computations were carried out as in Table 8, using none of the above short-cut procedures. It is easily seen, however, that the results are practically identical under both approaches.

Table 9 is also interesting in another connection. Except possibly for periods of catastrophe, the assumed survival trends have only rarely been exceeded over intervals of 10–20 years. Moreover, although similar or even more rapid trends over comparable periods may well occur in some underdeveloped areas during the next half century, it is unlikely that they can be sustained for any considerable length of time. This is especially true at the young ages, where a repetition of 12 per cent increases would soon lead to near-zero mortality. The assumed

Table 9. Per cent changes in age proportions at successive 15-year "censuses," under 12 per cent rise in 0–4 survivorship, linear rise in old-age survivorship up to 6 per cent, and two contrasting age distributions.[1]

Time	Age Group					
	0–14	15–29	30–44	45–59	60–74	75–89
(1)	(2)	(3)	(4)	(5)	(6)	(7)
	1. Age distribution in France, 1946					
15	0.3	−7.0	−8.8	−4.8	17.7	45.0
30	−0.3	−2.5	−11.2	−9.1	16.2	63.9
45	1.2	−2.8	−6.7	−11.3	11.2	60.6
60	1.6	−2.1	−7.5	−7.6	7.5	52.9
75	1.2	−1.8	−6.8	−8.6	9.9	47.1
90	1.0	−1.9	−6.2	−7.8	8.3	47.7
	2. Age distribution in Brazil, 1940					
15	3.1	−4.4	−6.3	−2.2	21.0	49.1
30	2.1	−0.2	−9.1	−7.0	19.0	67.8
45	2.6	−1.4	−5.3	−10.1	12.8	62.9
60	2.6	−1.0	−6.5	−6.7	8.7	54.5
75	2.3	−0.8	−5.8	−7.6	11.1	48.6
90	2.1	−0.8	−5.2	−6.8	9.5	49.3

[1] Derived in the same way as Table 8.

upper-age survival gains could conceivably continue for rather long periods, but their occurrence would presuppose a series of major revolutions in the care and prevention of old-age disease. Thus if the age-structure effects in Table 8 have been well above average for the past, those in Table 9 should rank among the highest to be expected at any time from short-run, "U-type" movements.

To illustrate the very nearly limiting effects that may result from movements resembling "reversed J's," we may assume a 12 per cent rise in 0–4 survivorship and a linear rise in upper-age gains extending up to 3 per cent. The increases by age from the first rise would therefore be twice the values shown in Table 3, while the further increases from the second would equal the ones described in (c) of Part III. Similarly, the increases in total numbers would be twice those in Table 4 plus the differences between that table and Table 7. The tabulation below presents some of the age-proportion changes:

Time	*0–14*	*15–29*	*30–44*	*45–59*	*60–74*	*75–89*
	1. Based on Age Distribution in France, 1946					
15	3.6	–4.0	–5.8	–3.8	7.9	22.0
45	5.4	1.1	–3.2	–9.9	1.0	26.4
75	4.7	1.2	–3.1	–6.9	0.6	15.9
	2. Based on Age Distribution in Brazil, 1940					
15	4.0	–3.6	–5.5	–3.4	8.3	22.5
45	3.8	–0.4	–4.7	–11.3	–0.6	24.5
75	3.1	0.0	–4.6	–8.3	–0.9	14.1

3. Thirdly, the techniques used in Part III for a single survival movement can readily be generalized in tracing the combined effects of successive changes. The increases by age and in total numbers after a series of changes are very nearly the sums of the increases that would be found if each survival movement occurred in isolation. Let I_T denote the increases by age that would result from the first change alone after T years (corresponding to any row in Table 6); let J_T denote the correspond-

ing increases that would occur T years after the second change; K_T the third set of increases, . . . As before, T has the values 15, 30, . . . , and the movements are assumed to occur at 15-year intervals. The successive movements may be of the same or different magnitudes; the main restrictions are that the survival trends are upward and that the 5–45 gains can be ignored. Then the increases by age would be given (very nearly) by I_{15} at time 15; by $I_{30} + J_{15}$ at time 30 (i. e., the increases at the various ages from the first change alone, 30 years after its occurrence, plus the corresponding increases from the second change alone, 15 years after its occurrence); by $I_{45} + J_{30} + K_{15}$ at time 45, and so forth. Similarly, the increases in toal population would be P_{15}, $P_{30} + Q_{15}$, $P_{45} + Q_{30} + R_{15}$, . . . , where P, Q, R, . . . denote the increases from I, J, K, . . . , respectively. For example, suppose we were dealing with a series of only two movements, each involving the same numerical changes as underlie Table 6. Then the increases by age at time 15 would be given by the first row of that table, those at time 30 by the sum of the first and second rows, those at time 45 by the sum of the second and third rows, and so forth. The rise in total numbers at these and later dates could be derived in exactly the same fashion from Table 7.[31] Such estimates would be approximate, since the accurate procedure would be to compound the ratios of successive survival rates (see footnote 13). But the errors would accumulate slowly unless a lengthy series of large changes were involved.

4. Finally, the effects of declining mortality on male age composition are likely to be very similar to those for females. Since the sex ratio at birth changes little over time, any increase in female births from improved female survivorship will be accompanied by a practically equal increase in male births. Thus, if

[31] Although no proof of these propositions is given here, their validity is not difficult to demonstrate. The reader can satisfy himself on this score by first considering two movements. It will be seen that, ignoring the question of compounding, the combined increase at each age and period would be the sum of two components: the gain (whether from increased survivors per birth or from increased births) resulting from the first movement plus the gain from the second. This implies that the effects on total numbers are similarly additive. Assuming next that the corresponding statements hold for any n changes, the same procedures can be followed to show that they would hold for n + 1 changes.

the former are rising by an additional 3 per cent per period, as in Part III, the latter will be rising at essentially the same rate. Also, the gains in young-age survival chances tend to be much the same for both sexes. The combination of these near equalities will generally result in highly comparable shifts in age proportions to about 60. Probably the only noteworthy differences that may be encountered in practice are at the upper ages, where female survival increases have often exceeded the gains for males. This possibility has not been given sufficient attention in the past, although it is especially interesting in connection with the "aging" of Western populations.

V

To sum up, the main links between mortality changes and age structure involve the following propositions:

1. The only measures of changing mortality which are appropriate for a general approach to the problem are proportionate or relative changes in survival rates. Because the usual emphasis in describing mortality trends has been on proportionate changes in mortality rates, rather than in their complements, this fundamental point has often been overlooked.[32]

2. The same point underlies the frequent finding in empirical studies that mortality movements tend to cause rather limited shifts in age distribution. A relative change in a survival rate is likely to be far below the relative change in the corresponding mortality rate.

3. The age-distribution effects of a given mortality movement are much more readily analyzed in relation to the distribution that would be found at successive periods without such movement, than in relation to the initial distribution. This distinction has been obscured by the tendency in the literature to deal with stable-age models, which have a constant distribution over time.

4. Year-to-year survival changes are of little interest, partly

[32] Nor is there any general way of applying measures of absolute change in mortality (or survival) rates, hence none for dealing with absolute changes in age proportions.

because their variations by age tend to be erratic, and partly because their cumulative effects on age structure are closely approximated by the average (relative) changes over longer intervals. Moreover, probably nothing very interesting can be said about survival downtrends, or average declines. These have been rare empirically and, like year-to-year changes, tend to vary in irregular fashion with age. For these reasons only uptrends are considered here.

5. The age patterns of survival increases (of a trend nature) have very typically resembled "L's," "reversed J's" or "near U's," the second being the most common. For practical purposes the changes between 5 and 45 can be assumed to be equal in each instance.

6. Proportionate increases in survival rates which are the same at all ages leave age composition unchanged. In essentials, therefore, the age-mortality problem is really to trace the effects of the larger increases to be expected below 5 and over 45. The combined influence of all age-specific survival movements may be regarded as the sum of the very nearly zero effects of the 5–45 changes and the effects resulting from the differences between these and the young-age and upper-age changes.

7. Additional increases at the young ages raise numbers by age and the total population in two ways: by increasing numbers of survivors per birth and by augmenting births (through the increased numbers in the reproductive years). Since age-specific fertility is assumed to be constant, both sources of increase have to be taken into account in tracing the effects of any young-age survival gain.

The analysis at this point is greatly simplified by several circumstances. The full (percentage) increase in any cohort, whether from added survivors per birth or from added births, is registered by about age 5 and remains the same at all later dates. Moreover, the increases from these sources can be combined by simple addition, with little loss of accuracy. Finally, the increases in births in successive periods begin to build up regularly after 30 to 45 years from the original mortality movement. At later dates the additional (percentage) rise in births per period is very nearly equal to half the rise in the 0–5 survival rate.

8. The effects of young-age survival gains can be further simplified, at least for general purposes, by considering females alone to begin with and by dealing with 15-year intervals of age and time. The reproductive span of life is more closely defined for females than for males and the female span effectively ends before upper-age survival gains (when they occur) become significant. In estimating the increases in female births, therefore, only the young-age gains need to be taken into account. The use of 15-year calendar intervals permits a convenient identification of the increases in the first and last "halves" of the female reproductive span of life, hence of the increased births contributed by each half. In turn, these increases are readily converted into increases in total births, since the ratio of births among women 15–30 or 30–45 to total births tends to remain fixed when age-specific fertility is constant.

9. In any given population, the effects of male young-age survival gains on male age structure are likely to be very nearly equal to the effects found for females. Since young-age gains are almost certain to be much the same for both sexes, the resulting increases in survivors per birth should correspond closely. The increases in births will also be very similar, owing to the near constancy of the sex ratio at birth.

10. The effects of upper-age survival gains on the numbers reaching age 45 can be ignored for both males and females. For practical purposes, the full influence of such gains in either case is simply to add to the increases that would have been found after 45 from the young-age gains alone. (The latter, of course, would not begin to affect the population over 45 until a nearly equal number of years had elapsed.) The combined effects of young-age and upper-age survival gains on the total size of population can be established in similar fashion, by adding the changes caused by each in isolation.

Thus where the upper-age gains for females are appreciably higher than for males, as is frequently the case, the differences first have to be taken into account at a late stage in the analysis.

11. To a close order of approximation, the transitional shifts in age proportions as a result of a single survival movement cease after 90 years; if we exclude the old ages, they cease after about 60 to 75 years. Thereafter the differences between the new proportions and the ones that would have been found under the previous mortality conditions remain very nearly constant. Thus only the first 60 to 90 years (four to six 15-year intervals) need be considered in order to establish the main transitional and long-run effects of a given survival trend.

12. Given a set of different 0–5 survival gains, the increases in numbers by age and total population which are caused by any one change can be used to estimate the increases caused by any other. It is a simple matter, therefore, to compare the age-proportion shifts arising from alternative numerical assumptions. The same is true of alternative upper-age changes, provided that these are rising more or less linearly beyond the 5–45 level.

13. Given a succession of survival movements (each of a trend nature), the combined (percentage) increases in numbers by age and in total population may be estimated by adding the increases, appropriately phased, that would be found from each movement alone. (For convenience, the movements may be assumed to occur at 15-year intervals.)

Finally, since the literature on the age-mortality problem has been largely concerned with Western populations, two further observations are worth making.

14. A given percentage decline in age-specific fertility would have practically no effect on age composition if accompanied by an equal percentage rise in young-age survivorship.[33] The "aging" of Western populations may therefore be looked upon

[33] In terms of the discussion in (b) of Part III, the combination of such changes would lead to zero increases in the survivors reaching age 4. The number at age 0 would be lowered by the extent of the fertility decline and the numbers at later ages would be smaller by decreasing margins up to 3. Since the population over 4 would be unaffected, the only increases that might arise would be from upper-age survival gains, as described in (c).

as the outcome of two broad tendencies. First, the percentage declines in Western fertility have been far greater than the relative increases in 0–5 survival rates. The net effects of these trends are the same as if the only movements were declines in early-age survivorship. Instead of the age proportions being raised in the younger years and then lowered, as in Table 5, the tendency in the West was to have early-age declines and upper-age increases. Secondly, these effects were often accentuated by the additional survival gains (over the 5–45 level) in the upper ages. Such gains have tended directly to augment the rise in upper-age proportions and, by raising total numbers, further to depress the young-age proportions. In other instances, when the age patterns of survival changes have been more or less L-shaped, upper-age movements have at least done nothing to offset the first of the above tendencies.

15. Assuming mortality declines continue in the West, their effects on age composition should be appreciably different from what they have been in the past. The maximum young-age survival gains still possible in Western populations are shown in Table 10. Since all are close to or below the 6 per cent level underlying Table 5, something like a 5 per cent rise in the proportion under 15 (or 3–4 per cent in the proportion under 30) is the most that can be expected from future trends. Moreover, this assumes practically zero survival gains at the upper ages. (If these were exactly zero, they would still be very close to the largest possible gains between 5 and 45.)

Considered by themselves, Western survival trends over the last century have showed a clear tendency to depress average age. In the future, however, the tendency should be strongly in the opposite direction. Indeed, as some recent trends suggest, age patterns resembling "J's" may well become frequent. To the extent that this is so, the effects on age structure would be analogous to those suggested by the differences between Tables 5 and 8.

As was stated at the outset, much of the recent interest in the age-mortality problem has come from the belated realization

Table 10. Maximum per cent increases still possible beyond recent age-specific survival rates in 13 Western populations, by sex.[1]

Age	Country[2]												
	Aust.	Bel.	Can.	Den.	Fr.	Gy.	Ire.	Neth.	N.Z.	Nor.	Sw.	Sz.	U.S.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
	1. Males												
0[3]	2.2	5.2	3.6	2.8	3.7	4.5	4.1	2.2	2.5	2.2	1.8	2.8	2.8
1	.3	.5	.3	.4	.6	.4	.8	.3	.2	.3	.2	.4	.2
2	.2	.3	.2	.2	.2	.2	.4	.2	.2	.2	.2	.2	.2
3	.2	.2	.2	.1	.1	.2	.3	.1	.1	.2	.1	.2	.1
4	.1	.2	.1	.1	.1	.2	.2	.1	.1	.1	.1	.1	.1
5	.1	.2	.1	.1	.1	.1	.2	.1	.1	.1	.1	.1	.1
10	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1
20	.2	.3	.2	.2	.2	.2	.3	.1	.2	.2	.2	.2	.2
30	.2	.4	.2	.2	.2	.2	.4	.1	.2	.2	.2	.2	.2
40	.3	.5	.3	.3	.5	.4	.5	.2	.3	.3	.3	.3	.4
50	.9	1.1	.9	.7	1.1	.9	1.0	.6	.7	.6	.6	.8	1.1
60	2.3	2.3	2.1	1.6	2.3	1.9	2.3	1.4	2.0	1.4	1.6	2.1	2.5
70	5.6	5.4	4.6	4.2	5.5	4.8	5.7	3.9	5.0	3.5	4.3	5.4	5.3
	2. Females												
0[3]	2.2	5.2	3.6	2.8	3.7	4.5	4.1	2.2	2.5	2.2	1.8	2.8	2.8
1	.3	.5	.3	.3	.5	.4	.8	.2	.2	.3	.2	.3	.2
2	.2	.2	.2	.2	.2	.2	.4	.2	.1	.2	.1	.2	.1
3	.1	.2	.1	.1	.1	.2	.3	.1	.1	.1	.1	.1	.1
4	.1	.2	.1	.1	.1	.1	.2	.1	.1	.1	.1	.1	.1
5	.1	.1	.1	.1	.1	.1	.2	.1	.1	.1	.1	.1	.1
10	.1	.1	.1	.0	.1	.1	.1	.0	.0	.0	.0	.0	.0
20	.1	.2	.1	.1	.1	.1	.3	.1	.1	.1	.1	.1	.1
30	.2	.2	.1	.2	.2	.2	.4	.1	.1	.1	.2	.1	.2
40	.3	.3	.3	.3	.3	.3	.5	.2	.2	.2	.2	.2	.3
50	.6	.6	.6	.6	.6	.6	.9	.4	.6	.4	.6	.5	.7
60	1.4	1.5	1.3	1.4	1.3	1.3	1.9	1.0	1.3	1.0	1.3	1.3	1.5
70	3.8	4.1	3.4	3.9	3.6	4.1	4.8	3.4	3.4	3.0	3.7	3.9	3.6

[1] Rates are for annual age intervals. All but infant survival rate (p_0) are taken from life tables in every case since 1945. Infant rates are from vital statistics for 1953 or 1954. Maximum increases defined as per cent differences between unity and rates.

[2] Countries are: Australia, Belgium, Canada, Denmark, France, Federal Republic of Germany, Ireland, Netherlands, New Zealand, Norway, Sweden, Switzerland, United States.

[3] Both sexes.

that Western mortality trends have typically slowed, not fostered, the aging of their populations. It is rather paradoxical to contemplate that the earlier conclusion, though based on incorrect reasoning, may yet be borne out by events.