# SOME DEMOGRAPHIC MEASUREMENTS FOR EGYPT BASED ON THE STABILITY OF CENSUS AGE DISTRIBUTIONS 

M. A. El-Badry ${ }^{1}$

Introduction

THERE is clearly some relation between the relative sizes of successive age groups in a population and the risks of death. When, for a considerable number of years, the population has been closed to migration and the age schedules of fertility and mortality have been relatively constant, the rate of growth of the population is also relatively constant and the proportional distribution of the population by age assumes a relatively fixed character known as "stable." These relations will be considered below. For the time being the essential point to note is that under such circumstances if the age of the population is accurately counted, it is a simple matter to compute the risks of death implicit in the age distribution.
It is the purpose of this paper to utilize the near stability of the Egyptian age distributions from 1907 to 1947 to obtain a life table and certain other measures of mortality and fertility. Before doing so, however, it will be necessary (1) to show that the age distribution is indeed approximately stable and (2) to correct the most obvious distortions in the census reporting of age. With this accomplished we can proceed to construct a life table representing the mortality conditions in Egypt during the period 1907-1947 and utilize the stability of demographic conditions to secure some measures of fertility as well.

The Stability of the Demographic Conditions in Egypt
If we confine ourselves to the crude birth rate as a measure of fertility in order to avoid the distortions which result from the erroneous age reporting of mothers, we find that this rate fluctuated haphazardly without any indication of a downward

[^0]trend between 1917 and 1946. The birth rate was 40.1 in 1917 and 42.7 in 1946 and the average rates in the intervals 19171926, 1927-1936, and 1937-1946 were 41.8, 43.6, and 41.7 respectively. It is true that the available birth rates are below reality because of under-reporting of births, especially in the areas without health bureaus; but it is also true that no indication of declining fertility can be traced in the incomplete data available. In fact, it seems very unlikely that there has been any considerable change in the people's customs regarding fertility, except perhaps among limited classes in the urban areas.

The situation is the same in the case of mortality, where the average crude death rates over the periods 1917-1926, 19271936, and 1937-1946-again deficient because of under-report-ing-have been constant and equal to about 27.0. ${ }^{2}$ (There has been a consistent downward trend in the death rates since 1946 but this is irrelevant to our discussion.)

The population can also be considered as "closed," since there has been virtually no emigration on the part of Egyptians and the movements of foreigners can have only a trivial effect on the age distribution of the population because, as can be seen from the accompanying table, they make up a very small percentage of total numbers.

| Year | No. of Foreigners | Proportion of Total <br> Population <br> Per Cent |
| :---: | :---: | :---: |
| 1927 |  | 1.6 |
| 1937 | 225,600 | 1.2 |
| 1947 | 186,515 | .8 |

The stability of fertility and mortality and the negligible amount of migration in the period 1907-1947 is also illustrated by the great similarity between the reported age distributions for both males and females as shown by Tables 1 and 2 and
${ }^{2}$ In computing the average for 1917-1926, the effect of the 1918 influenza epidemic was eliminated. The death rate rose from 29.4 in 1917 to 39.6 in 1918 and then dropped back to 29.4 in 1919.
Table 1. Population age distribution by decennial age intervals (in thousands).

| Year | 0-9 | 10-19 | 20-29 | 30-39 | 40-49 | 50-59 | 60-69 | 70-79 | 80-89 | 90-99 | $\begin{aligned} & 100 \text { and } \\ & \text { Over } \end{aligned}$ | Total |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A. males |  |  |  |  |  |  |  |  |  |  |  |  |
| 1907 | 1,664 | 1,200 | 816 | 740 | 509 | 688 |  |  |  |  |  | 5,617 |
| 1917 | 1,771 | 1,407 | 924 | 853 | 575 | 377 | 230 | 139 | 55 | 21 |  | 6,352 |
| 1927 | 1,937 | 1,545 | 1,099 | 991 | 664 | 392 | 245 | 120 | 47 | 15 | 3 | 7,058 |
| 1937 | 2,135 | 1,748 | 1,159 | 1,161 | 822 | 476 | 274 | 128 | 46 | 14 | 4 | 7,967 |
| 1947 | 2,495 | 2,132 | 1,367 | 1,283 | 1,000 | 594 | 337 | 132 | 40 | 12 |  | 9,392 |
| $1947{ }^{1}$ | 2,378 | 2,032 | 1,303 | 1,223 | 953 | 566 | 321 | 126 | 38 | 11 |  | 8,951 |
| B. females |  |  |  |  |  |  |  |  |  |  |  |  |
| 1907 | 1,678 | 942 | 918 | 757 | 515 | 763 |  |  |  |  |  | 5,573 |
| 1917 | 1,797 | 1,182 | 1,043 | 875 | 571 | 378 | 263 | 142 | 72 | 27 |  | 6,350 |
| 1927 | 1,964 | 1,338 | 1,234 | 1,017 | 657 | 411 | 275 | 138 | 64 | 18 | 4 | 7,120 |
| 1937 | 2,192 | 1,515 | 1,261 | 1,178 | 787 | 471 | 305 | 151 | 68 | 21 | 5 | 7,954 |
| 1947 | 2,505 | 1,995 | 1,498 | 1,348 | 985 | 624 | 382 | 162 | 58 | 18 |  | 9,575 |
| $1947{ }^{1}$ | 2,343 | 1,866 | 1,401 | 1,261 | 921 | 584 | 357 | 152 | 54 | 17 |  | 8,956 |

${ }^{1}$ Adjusted for over-reporting. (See text.)

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Table 2. Reported proportional age distributions.

| Year |
| :---: | 0-9

\footnotetext{
b. females

| 1907 | 3,011 | 1,690 | 1,647 | 1,358 | 924 | 1,369 |  |  |  |  |  | $\begin{gathered} 9,999 \\ 10,000 \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1917 | 2,830 | 1,861 | 1,643 | 1,378 | 899 | 595 | 414 | 224 | 113 |  |  |  |
| 1927 | 2,758 | 1,879 | 1,733 | 1,428 | 923 | 577 | 386 | 194 | 90 | 25 | 6 | 9,999 |
| 1937 | 2,756 | 1,905 | 1,585 | 1,481 | 989 | 592 | 383 | 190 | 85 | 26 | 6 | 9,998 |
| 1947 | 2,616 | 2,084 | 1,564 | 1,408 | 1,029 | 652 | 399 | 169 | 60 |  |  | 10,000 |



Fig. 1. Reported age distributions, by sex, 1907-1947.
Figures 1 and 2. Apart from a systematic improvement in the reporting of children within the age groups $0-9$ and $10-1$, which will be demonstrated later, and a systematic reduction in the over-estimation of old ages, the age distributions remain practically unchanged. They do exhibit, however, the effect of an unusually large cohort aged 0-9 in 1907 which maintains outstandingly high proportions from 1907 through 1947 among both males and females, as can be seen clearly from Table 2 and Figure 2. The fact that this cohort can be so readily identified in each of the five censuses itself tends to support the assumption of the stability of the age distribution throughout this period and testifies to the underlying accuracy of the census reports once they are corrected for systematic biases. In addition, the five censuses gave approximately stable growth rates during 1907-1937; the stability is broken only by the overenumerated 1947 census, as will be shown in the next section.


Fig. 2. Reported proportional age distributions, by sex, 1907-1947.
The fixed nature of mortality risks and the triviality of migration (two of three necessary components of a stable age distribution) can be seen from a simple measure like the ratio of the population aged 20 and over to the whole population 20 years earlier. (Age 20 was preferred to age 10 to avoid the aboveindicated systematic shift from the age group $0-9$ into the group 10-19.) These ratios are found to be practically constant:

| Year | Males | Females |
| :--- | :---: | :---: |
| 1927 | .637 | .685 |
| 1937 | .643 | .669 |
| $1947^{*}$ | .643 | .667 |

*The 1947 proportions were calculated after correcting for over-reporting in 1947 as will be shown in the following section.

One may conclude that there are substantial reasons for considering that the age distributions are essentially stable.

## Basis of the Correction Procedure

The distortion in the age distribution of a country can be repaired by statistical evidence secured either from the country itself or from other countries, or by a combination of such kinds of evidence.

The correction procedure adopted here is confined wholly to evidence obtained from Egyptian data. It is true that the corrected age distributions would have looked more satisfactory had external evidence also been utilized, but it was preferred to limit the corrections rather than include external experience in a study of mortality in Egypt.
Distortion has frequently been corrected by fitting a curve to the reported age distribution. This method has the drawback that the shape of the fitted curve is affected by the systematic errors in age reporting, such as tendencies to mention or avoid certain ages, under-enumeration of certain ages and overenumeration of others, and under-estimation or over-estimation of the individual's age.
Instead of this method, we shall apply here a procedure which secures indicators of necessary corrections from the five census age distributions along with evidence obtained from Egyptian statistics illustrating the practically unchanged mortality and fertility conditions in Egypt in 1907-1947 and the triviality of migration. This procedure will lead us to five essential corrections necessitated by the following systematic biases:
(1). Over-reporting in 1947.
(2). Erroneous reporting of children aged $10-19$ as being in the 0-9 age group.
(3). Erroneous reporting of males 20-29 as 10-19.
(4). Under-estimation of females aged 40-59.
(5). Over-estimation of old ages.

Adjustment of Over-Reporting in the 1947 Census
The 1947 census gave total population figures which consid-
erably exceeded expectations, especially in the case of the females. This can be seen from the table of growth rates:

| Interval | G.R. of Males | G.R. of Females |
| :---: | :---: | :---: |
| $1907-17$ | .012 | .013 |
| $1917-27$ | .011 | .011 |
| $1927-37$ | .012 | .011 |
| $1937-47$ | .016 | .019 |

This sharp increase in the 1947 growth rates can only be attributed to over-reporting since there is no evidence that it has been accompanied by changes in mortality, fertility, or migration. In fact, the large difference between the relative increases in the growth rates of females and males suggests the existence of over-reporting. This over-reporting was stimulated by a rationing census undertaken in 1945 which gave a total population of 24 million, or six and a half million over what one would expect on the basis of the 1927-1937 growth rate.

To adjust for this over-reporting, a continuation of the 1907-1937 growth rates, namely .01165 for males and .01186 for females, will be assumed here, giving a 1947 population of $8,951,000$ males and $8,956,000$ females-i.e. an over-reporting amounting to one million, or 5.6 per cent. It will also be assumed here that the degree of over-reporting is constant among the various age groups. There is certainly some error in this assumption, the effect of which would be negligible compared to the other distortions existing in the age distribution. The assumption was found to be satisfactory, however, in that it gave sex ratios in the various age groups which were consistent

Table 3. Reported sex ratios (males per female).

| Year | $0-9$ | $10-19$ | $20-29$ | $30-39$ | $40-49$ | $50-59$ | $60-69$ | $70-79$ | $80-89$ | 90 and Over |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1907 | .991 | 1.274 | .889 | .977 | .989 |  |  |  |  |  |
| 1917 | .986 | 1.190 | .886 | .975 | 1.007 | .997 | .875 | .979 | .764 | .778 |
| 1927 | .986 | 1.155 | .891 | .974 | 1.011 | .954 | .891 | .870 | .734 | .817 |
| 1937 | .974 | 1.154 | .919 | .986 | 1.044 | 1.011 | .898 | .848 | .676 | .692 |
| 1947 | .996 | 1.069 | .913 | .952 | 1.015 | .952 | .882 | .815 | .690 | .667 |
| 19471 | 1.015 | 1.089 | .930 | .970 | 1.035 | .969 | .899 | .829 | .704 | .647 |

${ }^{1}$ Adjusted for over-reporting. (See text.)


Fig. 3. Reported sex ratios, 1907-1947.
with the corresponding values calculated from the preceding censuses, as can be seen from the last row in Table 3 and from Figure 3. The slightly higher sex ratio in the group $0-9$ and lower sex ratio in the group $10-19$ in the adjusted census compared to the earlier censuses are due to the fact that the ratios given by the preceding censuses still need correction for erronous reporting of children aged $10-19$ as being in the $0-9$ group. The plausibility of the adjusted frequencies is also illustrated by the fact that the cohort survival rates of the various groups from 1937 to 1947 (Table 4) are much nearer to the averages of the corresponding rates in the preceding censuses than the high rates given by the reported 1947 data. This is a direct result of the reduction of the 1947 frequencies.

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Table 4. Reported cohort decennial survival rates.

| Interval | 0-9 | 10-19 | 20-29 | 30-39 | 40-49 | 50-59 | 60-69 | 70-79 | 80-89 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A. males |  |  |  |  |  |  |  |  |  |
| 1907-1917 | . 846 | . 770 | 1.045 | . 777 | . 741 |  |  |  |  |
| 1917-1927 | . 872 | . 781 | 1.073 | . 778 | . 682 | . 650 | . 522 | . 338 | . 327 |
| 1927-1937 | . 904 | . 750 | 1.056 | . 829 | . 717 | . 699 | . 522 | . 383 | . 383 |
| 1937-1947 | . 999 | . 782 | 1.107 | . 861 | . 723 | . 708 | . 482 | . 313 | . 196 |
| 1937-1947 ${ }^{1}$ | . 952 | . 745 | 1.055 | . 821 | . 689 | . 674 | . 460 | . 297 | . 196 |
| B. females |  |  |  |  |  |  |  |  |  |
| 1907-1917 | . 704 | 1.107 | . 953 | . 754 | . 734 |  |  |  |  |
| 1917-1927 | . 745 | 1.044 | . 975 | . 751 | . 720 | . 728 | . 525 | . 451 | . 306 |
| 1927-1937 | . 771 | . 942 | . 955 | . 774 | . 717 | . 742 | . 549 | . 493 | . 406 |
| 1937-1947 | . 910 | . 989 | 1.069 | . 836 | . 793 | . 811 | . 531 | . 384 | . 221 |
| 1937-1947 ${ }^{1}$ | . 851 | . 925 | 1.000 | . 782 | . 742 | . 758 | . 498 | . 358 | . 206 |

${ }^{1}$ Adjusted for over-reporting in 1947. (See text.)

Naturally the proportional age distribution, upon which most of the following discussion is based, will not be altered by this adjustment. The adjusted 1947 frequencies are shown in the last rows of Tables $1(A)$ and $1(B)$.

Correction of Female Frequencies Under 20 Years of Age
The discussion in this article is confined to ten-year age groupings in order to avoid some of the distortions resulting from the digit biases on the part of the respondents. The only available single-year age distribution, that of the 1927 census, reveals not only a concentration on ages ending with zero and five and a preference for even ages, but also outstanding popularity of certain middle ages. In that census the first ten most frequent ages were:

Males: $35,30,25,40,12,10,8,0,50,15$
Females: $30,25,40,35,20,50,8,10,0,12$
in which the ages of $35,30,25,40$ take leading positions and age 8 outnumbers all ages from 0 to 9 .
Now the female proportional age distributions [Table 2(B)] show that:
(a). The proportion of females under 20 years of age has been practically constant, ranging unsystematically between 46.4 per cent and 47.0 per cent.
(b). The proportion of the group 0-9 had a consistent downward trend while that of the group 10-19 maintained a consistent upward trend.
(c). These trends are toward reducing the distortion in the age distribution that existed at the mid-value of the group 10-19 from 1907 through 1937 and which has disappeared in 1947, as can be seen from Figure 2(B), in which the line for 1947 is free from the trough which existed at age 15 in the other censuses.
(d). Table 4(B) also shows cohort decennial survival rates ${ }^{3}$ which are too low for the group 0-9 and absurdly high rates for the group $10-19$, with a consistent upward trend in the rates of the first group accompanied by a consistent downward trend among those of the second group.

[^1]It is not possible to account for these relations in terms of decreasing fertility or increasing mortality, or increasing underenumeration of children under 10 . Therefore, one would be justified in assuming the existence of an increasingly accurate reporting of age from one census to the next, within the first two age groups. In this situation a sensible procedure would be to adopt the 1947 proportions in these groups as the most plausible.
To correct for this systematic improvement the excess in the $0-9$ female proportions over that of 1947 will be shifted into the $10-19$ group. This will be done for all of the censuses except 1907 which, as we noted above, has an extraordinarily large cohort $0-9$. In that case it is perhaps more plausible to allow for the cohort size as follows: (1) A straight line is fitted to the 1937, 1927, and 1917 reported proportions through the 1947 proportion; (2) the 1907 proportion is estimated from this straight line; (3) the excess of this estimated proportion over the 1947 proportion is then shifted into the $10-19$ group in 1907. The frequencies and proportions thus obtained will be as given in the first two columns of Tables 5(B) and 6(B).
As can be seen by comparing the first two columns of Tables $4(B)$ and $8(B)$, this correction improves the decennial survival rates of the first two age groups by increasing the rates of the $0-9$ group and decreasing those of the group 10-19.

The new proportions and frequencies straightened out the trough which existed at age 15 in both the frequency and the percentage curves and also were consistent with the 20-29 points on both curves. We will therefore not endeavor to undertake further correction in the 20-29 female group.

## Correction of Male Frequencies in the First Three Age Groups

The situation is more complicated in the case of the males. Here we notice again [Table 2(A) and Figure 2(A)] the existence of a downward trend in the proportions of the group 0-9 similar to that of the females and an upward trend in the group
Table 5. Corrected age distributions (in thousands).

| Year | 0-9 | 10-19 | 20-29 | 30-39 | 40-49 | 50-59 | 60-69 | 70-79 | 80-89 | 90-99 | 100 and Over | Total |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A. males |  |  |  |  |  |  |  |  |  |  |  |  |
| 1907 | 1,579 | 1,151 | 950 | 740 | 509 |  | - | - | 88 | - | $\square$ | 5,617 |
| 1917 | 1,694 | 1,345 | 1,063 | 853 | 575 | 377 | 230 | 139 | 55 |  | 1 | 6,352 |
| 1927 | 1,882 | 1,453 | 1,246 | 991 | 664 | 392 | 245 | 120 | 47 | 15 | 3 | 7,058 |
| 1937 | 2,125 | 1,643 | 1,274 | 1,161 | 822 | 476 | 274 | 128 | 46 | 14 | 4 | 7,967 |
| 1947 | 2,387 | 1,900 | 1,426 | 1,223 | 953 | 566 | 321 | 126 | 38 |  | 1 | 8,951 |
| b. females |  |  |  |  |  |  |  |  |  |  |  |  |
| 1907 | 1,508 | 1,112 | 918 | 753 | 519 |  |  |  | 63 |  |  | 5,573 |
| 1917 | 1,661 | 1,318 | 1,043 | 861 | 581 | 382 | 263 | 142 | 72 |  | 27 | 6,350 |
| 1927 | 1,863 | 1,439 | 1,234 | 1,002 | 672 | 411 | 275 | 138 | 64 | 18 | 4 | 7,120 |
| 1937 | 2,081 | 1,626 | 1,261 | 1,150 | 814 | 472 | 305 | 151 | 68 |  | 5 | 7,954 |
| 1947 | 2,343 | 1,866 | 1,401 | 1,226 | 956 | 584 | 357 | 152 | 54 |  | 17 | 8,956 |

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Table 6. Corrected proportional age distributions.

| Year |
| :---: | 0-9

b. females

| 1907 | 2,706 | 1,995 | 1,647 | 1,351 | 931 | 1,369 |  |  |  |  |  | $\begin{gathered} 9,999 \\ 10,001 \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1917 | 2,616 | 2,075 | 1,643 | 1,356 | 915 | 602 | 414 | 224 | 113 |  |  |  |
| 1927 | 2,616 | 2,021 | 1,733 | 1,407 | 944 | 577 | 386 | 194 | 90 | 25 | 6 | 9,999 |
| 1937 | 2,616 | 2,045 | 1,585 | 1,446 | 1,023 | 593 | 383 | 190 | 86 | 26 | 6 | 9,999 |
| 1947 | 2,616 | 2,084 | 1,564 | 1,369 | 1,067 | 652 | 399 | 169 | 60 |  |  | 9,999 |

10-19 interrupted only by the 1917 proportion which belongs to the above-mentioned large cohort aged $0-9$ in 1907. We also notice from Table 3 and Figure 3 that the reported sex ratios (males per thousand females) in the group 0-9 have been practically constant from 1907 to 1947, indicating that the improvement in the age reporting among the females $0-9$ was accompanied by a similar improvement among the males. The new sex ratios which result from carrying out the corrections among the females are:

| Year | $0-9$ | $10-19$ | $20-29$ |
| :---: | :---: | :---: | :---: |
| 1907 | 1.103 | 1.079 | .889 |
| 1917 | 1.066 | 1.068 | .886 |
| 1927 | 1.040 | 1.074 | .891 |
| 1937 | 1.026 | 1.074 | .919 |
| 1947 | 1.015 | 1.089 | .930 |

The downward trend in the $0-9$ group is due to the aboveindicated downward trend in the proportions of the males $0-9$ which has not yet been corrected. If this correction is carried out (by moving the surplus of 1947 into the $10-19$ group) the $10-19$ sex ratios will increase so much that it will be noticed immediately that there was mis-reporting of those aged 20-29 as being aged $10-19$. This erroneous reporting is also reflected by the low survival rates of the $10-19$ group and the impossibly high rates of the 20-29 group in Table 4(A).

Therefore there are two necessary corrections in the three age groups under consideration: (a). shifting a proportion of the male population aged $0-9$ into the $10-19$ group in order to correct for the downward trend in the proportions of the former group, (b). shifting a proportion from the $10-19$ group into the 20-29 group to correct for the wrong statement of ages 20-29 as 10-19.

Regarding the first correction, the 1947 proportion will be used again as a basis for correction after modifying it very slightly in order to maintain non-increasing sex ratios in the groups $0-9,10-19$, and 20-29 in 1947. This requires shifting a

| Year | $0-9$ | $10-19$ | $20-29$ | $30-39$ | $40-49$ | $50-59$ | $60-69$ | $70-79$ | $80-89$ | 90 and Over |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1907 | 1.047 | 1.035 | 1.035 | 1.035 | .983 | .981 |  |  |  |  |
| 1917 | 1.020 | 1.020 | 1.019 | 1.019 | .991 | .990 | .987 | .979 | .764 | .778 |
| 1927 | 1.011 | 1.010 | 1.010 | 1.010 | .989 | .988 | .954 | .870 | .734 | .817 |
| 1937 | 1.021 | 1.010 | 1.010 | 1.010 | 1.010 | 1.010 | 1.008 | .848 | .676 | .692 |
| 1947 | 1.019 | 1.018 | 1.018 | 1.018 | .998 | .997 | .969 | .829 | .704 | .647 |

Table 7. Corrected sex ratios.
frequency of 9,000 from the $10-19$ group into the $0-9$ group. This shift changes the $0-9$ proportion to .2667 rather than the reported .2657 . As in the case of the females, the excess over this proportion will be shifted from $0-9$ into $10-19$ for all the censuses except 1907, where it is preferred to allow for the large cohort size by following the same procedure used for the females.

The sex ratios in the groups $0-9$ and $10-19$ are then recalculated and the corrections in the 10-19 and 20-29 groups are carried out by requiring that, for each census, the sex ratio in the group 20-29 should not be higher than that in the group $10-19$. The result of this procedure is to shift certain frequencies from 10-19 into 20-29.

The resulting frequencies, proportions, and sex ratios in the three groups are shown in the first three columns of Tables 5 (A), 6(A), and 7 respectively.

One can check on the plausibility of the whole procedure by comparing the new survival rates as given by the first three columns of Table 8(A) with the reported ones [Table 4(A)]. The improvement on the low rates of the 10-19 group and the unreasonably high rates of the 20-29 group is quite obvious.

After carrying out these corrections we have the age distributions given in Tables 5(A) and 6(A) and Figures 4(A) and 5(A). The distributions appear to be reasonably satisfactory, despite the fact that the decennial survival rates of the cohorts 20-29 are still high.

Correction for Under-Estimation of Female Ages 40-59
Going back now to the sex ratios in the groups 30-39, 40-49, and 50-59 in Table 3, we see that these ratios, when compared

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Table 8. Corrected decennial survival rates.

| Interval | 0-9 | 10-19 | 20-29 | 30-39 | 40-49 | 50-59 | 60-69 | 70-79 | 80-89 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A. males |  |  |  |  |  |  |  |  |  |
| 1907-1917 | . 852 | . 924 | . 898 | . 777 | . 741 |  |  |  |  |
| 1917-1927 | . 858 | . 926 | . 932 | . 778 | . 682 | . 650 | . 522 | . 338 | . 327 |
| 1927-1937 | . 873 | . 877 | . 932 | . 829 | . 717 | . 699 | . 522 | . 383 | . 383 |
| 1937-1947 | . 894 | . 868 | . 960 | . 821 | . 689 | . 674 | . 460 | . 297 | . 196 |

B. females

|  |  |  |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $1907-1917$ | .874 | .938 | .938 | .772 | .736 |  |  |  |  |
| $1917-1927$ | .866 | .936 | .961 | .780 | .707 | .720 | .525 | .451 | .306 |
| $1927-1937$ | .873 | .876 | .932 | .812 | .702 | .742 | .549 | .493 | .406 |
| $1937-1947$ | .897 | .862 | .972 | .831 | .717 | .756 | .498 | .358 | .206 |



Fig. 4. Corrected age distributions, by sex, 1907-1947.
with those in the group 20-29 as given by column 4 of Table 7, reveal over-reporting of females $30-39$ as compared to the males and under-reporting of those in the $40-49$ group and, to a smaller extent, in the $50-59$ group. This suggests that the trough at age 35 in Figures 1(B) and 2(B) is in fact due to under-estimation of female ages 40-59.

To correct this distortion we are going to satisfy ourselves with the requirement that the sex ratios in the three groups-$30-39,40-49$, and 50-59-should not be increasing, and correct the female age distributions accordingly, thus getting the corrected data given in the corresponding columns of Tables 5(B), 6(B), 7, and 8(B) and also in Figures 4(B), 5(B), and 6. Again it should be stated that this correction is incomplete as shown by the fact that the decennial survival rates of the group 20-29 are still high, though more plausible than the reported ones.


Fig. 5. Corrected proportional age distributions, by sex, 1907-1947.
It is preferred to terminate the corrections at this stage of very essential corrections rather than go further into a more elaborate correction procedure. Regarding the old ages, Table 2 and Figure 2 indicate a systematic improvement over time in the over-estimation of ages 70 and over for both males and females. We can therefore partially correct this over-estimation by adopting the 1947 proportions in the tail of the age distribution starting from age 60. This will be done later when we construct a model age distribution for the interval 19071947.

## Life Tables for Egypt

Attempts to construct life tables for Egypt started in 1936 when El-Shanawany, (1), constructed the "First National Life Tables for Egypt" for the period 1917-1927. Confronted


Fig. 6. Corrected sex ratios, 1907-1947.
with unreliable death registration and age reporting at death, El-Shanawany used the method of comparing the age distributions at two censuses, of which the underlying theory is that if $P_{x}^{t}$ is the population aged $x$ at time $t$ and if, after an interval of length $a, P_{x+a}^{t+a}$ is the population aged $x+a$ then, in the absence of migration and assuming that $\mathrm{P}_{x}^{t}$ and $\mathrm{P}_{x+a}^{t+a}$ are accurate or subject to the sam 2 degree of error, $\mathrm{P}_{\mathrm{x}+\mathrm{a}}^{\mathrm{t}+\mathrm{a}}$ would be the survivors of $P \frac{t}{x}$ after the interval $t$ and, consequently, ${ }_{2} p_{x}$ the probability of surviving a years for an individual aged x would simply be the quotient $\frac{P_{x+a}^{t+a}}{P_{x}^{t+a}}$. The method has also been applied for the construction of Indian life tables, (4). The application of this method is generally preceded by a process of smoothing the age distributions at the two censuses under consideration because the age distributions under the circumstances where the method is applicable are usually distorted by erroneous age reporting and probably by under-enumeration.

El-Shanawany used the Pearson Type ix curve to smooth the data. Pearson Types ix and I have been applied to Indian census age distributions.
Another life table for Egypt was constructed by Kiser, (2), for the period 1927-1937. His procedure was similar to ElShanawany's with the addition of some correction for underenumeration of children under 5 and redistribution of the population in the other age groups in order to reduce some of the more prominent irregularities. The latter amendment consisted chiefly in systematically consigning given proportions of persons reporting ages divisible by five to the preceding five-year group. The single-year proportions within every age group of the 1927 census were applied to the 1937 age groupings.
A different line was followed by Abdel Rahman, (3), in constructing the 1936-1938 life tables for Egypt. He applied the mathematically rigorous method of comparing deaths in an age group with the population in that group. He followed mainly King's method which is used in constructing British life tables.
The application of the census differencing method is subject to the drawback that, since the smoothing is done by curve fitting which supplies the "best" possible fit to the distorted census data, the smooth curves may not maintain adequate representation of the correct age distributions, thus giving rise to errors in ${ }_{\mathrm{a}} \mathrm{p}_{\mathrm{x}}$ resulting from the estimation errors in $\mathrm{P}_{\mathrm{x}}^{\mathrm{t}}$ and $P_{x+a}^{t+a}$ and becoming more serious when the latter errors are in opposite directions. This is reflected in the fact that in the two attempts (1) and (2) the differences between the estimated $P_{x}^{t}$ and $P_{x+a}^{t+a}$ were too small in the ages below 10 to give rise to reasonable values of $p_{x}$, thus obliging the two writers to discard these estimates of $p_{x}$. El-Shanawany, (1), started his life table at age 10 and Kiser, (2), reconstructed ${ }_{1} q_{0},{ }_{4} q_{1},{ }_{5} q_{5}$ from the age-specific mortality rates in the health bureau areas.

The mathematically rigorous method applied in (3), used with various modifications in all countries that have reliable mortality data, has the drawbacks-when applied to Egypt-
of relying heavily on the frequencies and age distributions of deaths and also on a mathematically smoothed census age distribution. The death reporting in the country is far from adequate and varies according to whether or not the areas have health bureaus and also according to the distance between an area and the bureau to which it belongs. To mention an example, the following table is quoted from a report, (5), on a health survey undertaken in a rural area 30 kilometers north of Cairo, comprising a number of villages which are essentially the same in physical and environmental characteristics. The third column in the table gives the crude death rates in the villages in 1948 as calculated from the records of the health bureau and the fourth column gives the death rates in 1951 as calculated from the returns of the survey. The deficiency of the former rates and the dependence of this deficiency on the distance from the bureau are obvious.

| Village | Distance from <br> Health Bureau <br> in Kilometers | Crude Death Rates <br> Per 1,000 |  |
| :--- | :---: | :---: | :---: |
|  |  | 1948 | 1951 |
| Sindbis | 0 | 32 | 38 |
| Quaranfil | 2 | 19 | 33 |
| Barada | 2 | 23 | 28 |
| Aghour El-Sughra | 6 | 12 | 29 |

These deficiencies in the death rates would certainly inflate the $p_{x}$ 's and the life expectation in the life table.

The validity of the results of this method also depends upon the accuracy of the age reporting of the deceased which has the same sources of distortion as the census age distribution.

Apart from the above drawbacks inherent in the two methods that have been applied in life table construction for Egypt, the methods have the disadvantage of being techniques which are too elaborate in view of the accuracy of the utilized data. Having in mind that the results of such procedures can at best be approximate when applied to distorted and deficient data, it seems more appropriate to look for a method which is admit-
tedly approximate but at the same time has a simple procedure that suits the quality of the available data. This is the aim of the following attempt which is based upon the approximate stability of the age distributions.

## A Life Table Based on the Notion of Stable Population

As pointed out by Lotka (6), if mortality and fertility in a closed population remain unchanged then the age distribution of the population will eventually reach a stable stage, called the stable age distribution, regardless of the initial age distribution and the initial mortality and fertility rates. The proportion of the population aged $x$ will be

$$
\begin{equation*}
\mathrm{c}(\mathrm{x})=\mathrm{b} \mathrm{e}^{-\mathrm{rx} \frac{\mathrm{l}_{\mathrm{x}}}{l_{0}},} \tag{1}
\end{equation*}
$$

where b is the birth rate of the eventual stable population, $r$ is the rate of increase of that population, $l_{x}$ the number of individuals surviving to age x out of a cohort $\mathrm{l}_{0}$ individuals starting life and going through the mortality experience of the population. In other words, $\frac{l_{x}}{l_{\mathrm{x}}}$ is the probability that an individual will attain the age x . If the stable age distribution and the rate of increase r are known, then (1) gives the relationship

$$
\begin{equation*}
\frac{l_{x_{2}}}{l_{x_{1}}}=\frac{c\left(x_{2}\right) e^{r x_{2}}}{c\left(x_{1}\right) e^{r x_{1}}} \tag{2}
\end{equation*}
$$

which enables us to calculate the survivors $l_{x}$ at any age $x$. Once the l's are available, we can calculate the probabilities of surviving a year for an individual aged x from the relation

$$
\begin{equation*}
{ }_{a} p_{x}=\frac{l_{x+a}}{l_{x}}, \tag{3}
\end{equation*}
$$

and consequently construct the whole life table.
Now since the mortality and fertility conditions in Egypt in the period 1907-1947 remained practically unchanged and since the age distributions during that period were quite similar, and having in mind that the population was virtually "closed,"
we can assume that the age distribution in that period was practically stable and thus use the relationship (2) to construct a life table representing the mortality conditions in Egypt in the same period. ${ }^{4}$

The calculation process will consist of the following steps:
(1). Calculating the stable age distribution. This will be done here by averaging the proportion age distributions which we assume to be approximately stable.
(2). Estimating the proportions c (x) of the population at any desired pivotal points either by simple methods like taking one-tenth of the proportion in a decennial age to represent the proportion at the middle of that group, or taking the average of two adjacent groups as an estimate of the proportion at their mid-point, or by a more refined method of interpolation.
(3). Evaluating the rate of increase r from the populations under consideration. The rates which will be applied here are those calculated for the period $1907-1937$ by applying the Malthusian population formula

$$
\begin{equation*}
N_{t}=N_{o} e^{r t} \tag{4}
\end{equation*}
$$

which gives rates of increase equal to .01165 for males and .01186 for females.
(4). To get the $l_{x}$ column of the life table, the exponential $\mathrm{e}^{\mathrm{rx}}$ is then calculated at the pivotal points and multiplied by the estimated $c(x)$ to give $l_{x}^{\prime}=\frac{b}{I_{o}} l_{x}$ at the same points. $Y_{x}^{\prime}$ at any other desired points can then be calculated by interpolation.
(5). Once we have $\mathrm{l}_{x}^{\prime}$ we can calculate ${ }_{\mathrm{a}} \mathrm{p}_{\mathrm{x}}$ from the formula

$$
\begin{equation*}
{ }_{a} p_{x}=\frac{l_{x+a}^{\prime}}{l_{x}^{\prime}} \tag{5}
\end{equation*}
$$

and consequently fill in the ${ }_{a} p_{x}$ and ${ }_{a} q_{x}$ columns of the life table.

[^2](6). ${ }_{a} L_{x}^{\prime}=\frac{b}{1_{0}} L_{x}=\frac{b}{1_{0}} \int_{x}^{x+a} 1_{x} d_{x}$, which is proportional to the stationary life table population aged x to $\mathrm{x}+\mathrm{a}$ is obtained from the $l_{x}^{\prime}$ values either by simple averaging or by simple or refined numerical integration. $T_{x}^{\prime}=\frac{b}{l_{o}} T_{x}=\frac{b}{l_{0}} \int_{x}^{\infty} l_{x} d x$, which is proportional to the number of years of life remaining to survivors at age x , can be calculated by summing ${ }_{a} \mathrm{~L}_{x}^{\prime}$ or by numerical integration of $\mathrm{l}_{\mathrm{x}}$.
(7). Finally, the life expectation at age $\mathrm{x}, \mathrm{e}_{\mathrm{x}}=\frac{\mathrm{T}_{x}}{1_{x}}$, will result from dividing $\mathrm{T}_{x}^{\prime}$ by $\mathrm{l}_{\mathrm{x}}^{\prime}$.

It is obvious that this method can be applied to countries and time intervals only if the assumption of an approximately stable population is justified. This situation is likely to arise in underveloped areas where the high mortality and fertility have not yet started to decrease and where the population does not suffer considerably from epidemics, wars, migration, or other disrupting factors.

The simplicity of the method is also very clear. This is an essential requisite of a method applied to data whose distortions and deficiencies do not justify the application of more theoretically refined methods and heavy calculations. As a matter of fact, given two census age distributions or one census and the growth rate, and even without attempting to correct the distortions in the age distributions, we can very quickly calculate the $l_{x}$ 's and the life expectations at the $x$ 's where the age distribution does not appear to be distorted. Going back to the reported census data of Egypt for the decennial age intervals 1907-1947 (Table 2) and calculating the average proportional age distribution without any corrections for overenumeration in 1947, taking these averages as proportional to $c(x)$ at the mid-values of the decennial intervals, giving $r$ the approximate value of .012 for both males and females and taking ${ }_{10} L_{x}=5\left(l_{x}+l_{x+10}\right)$, we get the following rough estimates for $l_{x}^{\prime}$ 's and $\dot{e}_{x}^{\prime}$ 's at ages $5,35,45$, and 55:

| Age | Males |  | Females |  |
| :---: | :---: | :---: | :---: | :---: |
|  | $1_{x}$ | $\stackrel{\circ}{e}$ | $1_{x}$ | ${ }^{\circ}{ }_{x}$ |
| 5 | 10,000 | 42.4 | 10,000 | 42.4 |
| 35 | 7,138 | 25.3 | 7,242 | 25.8 |
| 45 | 5,667 | 20.6 | 5,512 | 22.4 |
| 55 | 3,918 | 17.5 | 3,943 | 19.3 |

which, in spite of the crude and rapid method followed in their estimation, do not differ considerably from those obtained by more elaborate techniques, as can be seen by comparing them with their corresponding values in Figures 7 and 8.

To calculate a complete life table it is necessary, however, to correct the reported age distributions. The corrected age distributions in Table 6 are found adequate, despite the fact that the proportions in the $30-39$ group are still high. When calculating the average proportional age distribution it was preferred to use the tail of the 1947 distribution from age 60 upward as the tail of the average distribution in order to utilize what is apparently a systematic improvement in the overestimation of old ages from one census to the next, as indicated before. The average proportional age distributions are as follows: ${ }^{5}$

| Age Group | Male | Female |
| :---: | :---: | :---: |
| $0-9$ | 2,713 | 2,653 |
| $10-19$ | 2,095 | 2,059 |
| $20-29$ | 1,674 | 1,646 |
| $30-39$ | 1,385 | 1,396 |
| $40-49$ | 976 | 983 |
| $50-59$ | 599 | 611 |
| $60-69$ | 361 | 402 |
| $70-79$ | 142 | 170 |
| $80-89$ | 42 | 60 |
| $90-99$ | 10.5 | 15.3 |
| $100 \&$ Over | 2.5 | 3.7 |
| Total | 10,000 | 9,999 |

[^3]

Fig. 7. Survivorship ( $\mathrm{l}_{\mathrm{x}}$ ) values ( $\mathrm{l}_{10}=10,000$ ) as given by life tables for Egypt, by sex, selected dates.
In the calculation of the proportions at the pivotal ages it was preferred, in order to get rid of the inflation in the group 30-39 and at the same time avoid further correction of the data, to estimate the proportion of the population at ages ending with zero by averaging the proportions in the two adjacent age groups. While this method smoothed out the inflation at the mid-value of the 30-39 group, it proved to be as accurate as concentrating the whole proportion in one decennial age group at the mid-value of that group because the $l_{x}$ curve resulting from the former method was found to be practically identical with that given by the latter method at all ages ending with zero for both males and females. To extend the life table to age 5, one-tenth of the proportion in the first age group (0-9) was taken as an estimate of the proportion at age 5 .

The resulting $\mathrm{l}_{5}^{\prime}, \mathrm{l}_{10}^{\prime}, \mathrm{l}_{20}^{\prime}, \ldots$ are transformed into $\mathrm{l}_{5}, \mathrm{l}_{10}, \mathrm{l}_{20}$, $\ldots$ by taking $l_{5}=10,000$ and calculating $l_{10}, l_{20}, \ldots$ accordingly. The calculations are as follows:

Some Demographic Measurements for Egypt

| Pivotal <br> Agex | Males |  |  |  | Females |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | c (x) | e. 01165 x | $1^{\prime} \times$ | $1_{x}$ | c ( x ) | e.01186x | $1^{\prime} \times$ | $1_{\text {x }}$ |
| 5 | 271.3 | 1.0599 | 287.6 | 10,000 | 265.3 | 1.0611 | 281.5 | 10,000 |
| 10 | 240.4 | 1.1236 | 270.1 | 9,392 | 235.6 | 1.1259 | 265.3 | 9,425 |
| 20 | 188.5 | 1.2624 | 238.0 | 8,275 | 185.3 | 1.2677 | 234.9 | 8,345 |
| 30 | 153.0 | 1.4184 | 217.0 | 7,545 | 152.1 | 1.4273 | 217.1 | 7,712 |
| 40 | 118.1 | 1.5936 | 188.2 | 6,544 | 119.0 | 1.6070 | 191.2 | 6,792 |
| 50 | 78.8 | 1.7905 | 141.1 | 4,906 | 79.7 | 1.8094 | 144.2 | 5,123 |
| 60 | 48.0 | 2.0117 | 96.6 | 3,359 | 50.7 | 2.0372 | 103.3 | 3,670 |
| 70 | 25.2 | 2.2603 | 57.0 | 1,982 | 28.6 | 2.2938 | 65.6 | 2,330 |
| 80 | 9.2 | 2.5396 | 23.4 | 814 | 11.5 | 2.5826 | 29.7 | 1,055 |
| 90 | 2.63 | 2.8534 | 7.5 | 261 | 3.77 | 2.9078 | 11.0 | 391 |
| 100 | . 65 | 3.2059 | 2.1 | 73 | . 95 | 3.2740 | 3.1 | 110 |

From the $l_{x}$ 's we get the ${ }_{a} p_{x}$ 's by consecutive divisions, and hence the ${ }_{a} q_{x}$ 's. $T_{x}$ for ages between 10 and 70 is then calculated by the approximate Reed-Merrell formula (quoted from (7)).

Fig. 8. Life expectancies as given by life tables for Egypt, by sex, selected dates.


| $x$ | Males |  |  | Females |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $1_{x}$ | apx | $\stackrel{\circ}{x}$ | $1_{x}$ | apx | $\stackrel{\circ}{x}$ |
| 5 | 10,000 | . 9392 | 43.2 | 10,000 | . 9425 | 45.0 |
| 10 | 9,392 | . 8811 | 40.8 | 9,425 | . 8854 | 42.6 |
| 20 | 8,275 | . 9118 | 35.7 | 8,345 | . 9241 | 37.5 |
| 30 | 7,545 | . 8673 | 28.7 | 7,712 | . 8807 | 30.1 |
| 40 | 6,544 | . 7497 | 22.2 | 6,792 | . 7543 | 23.5 |
| 50 | 4,906 | . 6847 | 18.0 | 5,123 | . 7164 | 19.5 |
| 60 | 3,359 | . 5901 | 14.0 | 3,670 | . 6349 | 15.3 |
| 70 | 1,982 | . 4107 | 10.3 | 2,330 | . 4528 | 11.2 |
| 80 | 814 | . 3206 | 7.9 | 1,055 | . 3706 | 8.6 |
| 90 | 261 | . 2797 | 5.2 | 391 | . 2813 | 5.6 |
| 100 | 73 |  |  | 110 |  |  |

Table 9. Abridged life table for Egypt, 1907-1947.

$$
\begin{equation*}
\mathrm{T}_{\mathrm{x}}=4.1667 \mathrm{l}_{\mathrm{x}}+.8333 \mathrm{l}_{\mathrm{x}+10}+10{ }_{a}^{\infty}{\underset{1}{5} \mathrm{l}_{\mathrm{x}+10 a}}^{\infty} \tag{6}
\end{equation*}
$$

For ages 80 and $90, \mathrm{~T}_{\mathrm{x}}$ is taken to be equal to the area under the second degree parabola through $\mathrm{l}_{80}, \mathrm{l}_{90}$, and $\mathrm{l}_{100}$ between these ages and 100 . We thus find that

$$
\begin{equation*}
\mathrm{T}_{80}=\frac{10}{3}\left(\mathrm{l}_{80}+4 \mathrm{l}_{80}+\mathrm{l}_{100}\right) \tag{7}
\end{equation*}
$$

and

$$
\begin{equation*}
\mathrm{T}_{90}=\frac{5}{6}\left(-\mathrm{l}_{80}+8 \mathrm{l}_{90}+5 \mathrm{l}_{100}\right) \tag{8}
\end{equation*}
$$

To get ${ }_{5} \mathrm{~T}_{\mathrm{x}}$ we calculate ${ }_{5} \mathrm{~L}_{5}$ and add it to ${ }_{10} \mathrm{~T}_{\mathrm{x}}$. Since the errors in calculating the area under the $l_{x}$ curve between ages 5 and 10 would have a negligible effect on $\dot{\dot{~}}_{5}$, the simple estimate ${ }_{5} L_{5}=\frac{5}{2} \quad\left(l_{5}+l_{10}\right)$ seems to be quite satisfactory (an estimate equal to the area under the parabola through the $l_{5}, l_{10}$, and $l_{20}$ points, between ages 5 and 10 , which will be given by ${ }_{5} L_{5}=\frac{l}{36}\left(16 l_{5}+21 l_{10}-l_{15}\right)$ gave an almost identical result $)$.

The resulting life table is given in Table 9.

## Constructing a Complete Life Table

(a). Life Table Values at Quinquennial Ages. A life table
with five-year age intervals can be readily calculated from the $1_{x}$ column of Table 9 by interpolating for the $l_{x}$ 's at ages ending with 5. The interpolation is carried out for all ages from 25 through 85 by fitting a second degree moving parabola to four decennial $l_{x}$ 's, two on each side of the required age. ${ }^{6}$ The interpolation formula will be:

$$
\begin{equation*}
l_{1+5}=\frac{9}{16}\left(l_{1}+l_{1+10}\right)-\frac{1}{16}\left(l_{1-10}+l_{1+20}\right) \tag{9}
\end{equation*}
$$

where $i=20,30, \ldots 80$.
To get $l_{15}$ a second degree parabola is fitted to $l_{5}, l_{10}, l_{20}$, and $1_{25}$. This curve gives

$$
\begin{equation*}
l_{15}=\frac{2}{3}\left(l_{10}+l_{20}\right)-\frac{1}{6}\left(l_{5}+l_{25}\right) \tag{10}
\end{equation*}
$$

$\mathrm{I}_{95}$ is then obtained by fitting a second degree parabola to the logarithms of $\mathrm{I}_{70}, \mathrm{I}_{80}, \mathrm{I}_{90}$, and $\mathrm{l}_{100}$ which leads to the formula

$$
\begin{equation*}
\log 1_{95}=-\frac{9}{80} \log 1_{70}+\frac{17}{80} \log 1_{80}+\frac{33}{80} \log 1_{90}+\frac{39}{80} \log 1_{100} \tag{11}
\end{equation*}
$$

Having obtained the quinquennial $l_{x}$ 's, $T_{x}$ can be calculated for all quinquennial ages from 10 to 90 by means of the ReedMerrell formula which we quote from (7),

$$
\begin{equation*}
\mathrm{T}_{\mathrm{x}}=-.20833 \mathrm{l}_{\mathrm{x}-\mathrm{5}}+2.5 \mathrm{l}_{\mathrm{x}}+.20833 \mathrm{l}_{\mathrm{x}+5}+5 \sum_{\alpha=1}^{\infty} \mathrm{l}_{\mathrm{x}}+5 \alpha, \tag{12}
\end{equation*}
$$

and $T_{90}$ and $T_{95}$ are obtained, as before, by fitting a second degree parabola to $l_{90}, l_{95}$ and $l_{100}$ in which case formulas 7 and 8 become

$$
\begin{equation*}
\mathrm{T}_{90}=\frac{5}{3}\left(\mathrm{l}_{90}+4 \mathrm{l}_{95}+\mathrm{l}_{100}\right) \tag{13}
\end{equation*}
$$

and

$$
\begin{equation*}
\mathrm{T}_{95}=\frac{5}{12}\left(-\mathrm{l}_{90}+8 l_{95}+5 l_{100}\right) \tag{14}
\end{equation*}
$$

(b). Life Table Values at Ages Zero and One. To extend the life table backward to ages one and zero it is not possible to follow the procedure which has been adopted so far, namely

[^4]calculating $c(x)$ and $\mathrm{l}_{\mathrm{x}}$ and from this $\mathrm{p}_{\mathrm{x}}$ and $\dot{e}_{x}$, because the proportions of children in the first five years of life cannot be calculated accurately from the census data. Kiser, (2), handled this point by calculating ${ }_{1} q_{0},{ }_{4} q_{1}$, and ${ }_{5} q_{5}$ from the vital statistics of the health bureau areas. In Indian life tables, as pointed out in (4), $l_{x}$ values are projected for ages below 5 by fitting the curve $\mathrm{I}_{\mathrm{x}}=\mathrm{A}+\mathrm{Hx}+\mathrm{BC}^{\mathrm{x}}+\frac{\mathrm{m}}{\mathrm{nx}+1}$.

Owing to the distortions and inadequacies in the data that have to be used for calculating ${ }_{1} q_{0},{ }_{4} q_{1}$, it was preferred here to estimate upper and lower limits for these two values. No claim can be made, however, that the following estimates stand far above the level of "reasonable guessing."
In 1946 the infant mortality rates per live birth calculated from vital statistics were .1906 for males and .1834 for females. These rates can be used as minima for ${ }_{1} q_{0}$ because the underreporting of infant deaths is heavier than that of births, as indicated by Weir, (5). According to Table 32 of Weir's report, the reported infant rates are deficient by about 26 per cent. If we consider this as a maximum rate of deficiency, since Weir's survey area was utterly rural, we get as maximum values of ${ }_{1} \mathrm{q}_{0}: .2402$ for males and .2311 for females.

To estimate limits for ${ }_{4} q_{1}$ we had to calculate first ${ }_{4} m_{1}$ by dividing the deaths $1-4$ obtained from the vital statistics of 1946 by estimates of the population $1-4$ in the middle of the same year. This procedure gave ${ }_{4} \mathrm{~m}_{1}$ equal to .0641 for males and .0607 for females, which we adopt as lower limits owing to the deficiencies in death reporting. To calculate an upper limit of ${ }_{4} \mathrm{~m}_{1}$ it is necessary to estimate the degree of under-reporting of deaths in these ages. We have to be satisfied here with an estimate calculated from crude death rates given in Table 32 of Weir's report and belonging to dates before and after his survey. This estimated degree of deficiency amounts to 30 per cent. We then inflate the above ${ }_{4} \mathrm{~m}_{1}$ by this percentage to get an upper limit of ${ }_{4} \mathrm{~m}_{1}$ which will be .0833 for males and .0789 for females. Finally the Reed-Merrell equation

$$
\begin{equation*}
{ }_{4} \mathrm{q}_{1}=1-\exp \left\{-4{ }_{4} \mathrm{~m}_{1}\left(.9806-2.079{ }_{4} \mathrm{~m}_{1}\right)\right\} \tag{15}
\end{equation*}
$$

was used to get the following estimates of ${ }_{4} \mathrm{q}_{1}$
lower limits: . 1952 for males and .1873 for females
upper limits: .2359 for males and .2272 for females
The life table is then completed by calculating the limits of $l_{0}$ and $l_{1}$ which would end in $l_{5}=10,000$ after going through the mortality experience expressed by the above limits of ${ }_{1} q_{0}$ and ${ }_{4} q_{1}$. The limits of $l_{0}$ and $l_{1}$ will be:

Males
Females

|  | Upper | Lower | Upper | Lower |
| :--- | :--- | :--- | :--- | :--- |
|  | Limit | Limit | Limit | Limit |
| $\mathrm{l}_{0}$ | 17,224 | 15,351 | 16,829 | 15,069 |
| $\mathrm{l}_{1}$ | 13,087 | 12,425 | 12,940 | 12,305 |

Table 10. Life table for Egypt, 1907-1947, by quinquennial age intervals.

| $x$ | Males |  |  | Females |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $1_{x}$ | $\mathrm{apx}^{\text {a }}$ | ${ }^{\circ}{ }_{x}$ | $1_{x}$ | apx | $\stackrel{\circ}{x}$ |
| 0 | $(17,224)$ | (.8094) | (31.8) | $(16,829)$ | (.8166) | (33.6) |
|  | $(15,351)$ | (.7598) | (28.5) | $(15,069)$ | (.7689) | (30.2) |
| 1 | $(13,087)$ | (.8048) | (38.2) | $(12,940)$ | (.8127) | (40.0) |
|  | $(12,425)$ | (.7641) | (36.4) | $(12,305)$ | (.7728) | (38.1) |
| 5 | 10,000 | . 9392 | 43.2 | 10,000 | . 9425 | 45.0 |
| 10 | 9,392 | . 9366 | 40.8 | 9,425 | . 9384 | 42.6 |
| 15 | 8,797 | . 9407 | 38.4 | 8,844 | . 9436 | 40.2 |
| 20 | 8,275 | . 9550 | 35.7 | 8,345 | . 9608 | 37.5 |
| 25 | 7,903 | . 9547 | 32.3 | 8,018 | . 9618 | 33.9 |
| 30 | 7,545 | . 9412 | 28.7 | 7,712 | . 9488 | 30.1 |
| 35 | 7,101 | . 9216 | 25.3 | 7,317 | . 9282 | 26.6 |
| 40 | 6,544 | . 8800 | 22.2 | 6,792 | . 8821 | 23.5 |
| 45 | 5,759 | . 8519 | 19.9 | 5,991 | . 8551 | 21.3 |
| 50 | 4,906 | . 8390 | 17.9 | 5,123 | . 8540 | 19.5 |
| 55 | 4,116 | . 8161 | 15.9 | 4,375 | . 8389 | 17.4 |
| 60 | 3,359 | . 7880 | 13.9 | 3,670 | . 8144 | 15.2 |
| 65 | 2,647 | . 7488 | 12.0 | 2,989 | . 7795 | 13.1 |
| 70 | 1,982 | . 6796 | 10.2 | 2,330 | . 7082 | 11.1 |
| 75 | 1,347 | . 6043 | 8.9 | 1,650 | . 6394 | 9.7 |
| 80 | 814 | . 5848 | 8.1 | 1,055 | . 6265 | 8.8 |
| 85 | 476 | . 5483 | 7.3 | 661 | . 5915 |  |
| 90 | 261 | . 5441 | 5.8 | 391 | . 5448 | 5.8 |
| 95 | 142 | . 5141 | 3.6 | 213 | . 5164 | 3.6 |
| 100 | 73 |  |  | 110 |  |  |

from these values upper and lower limits of $\mathrm{L}_{0}$ and ${ }_{4} \mathrm{~L}_{1}$ (and consequently $\mathrm{T}_{1}$ and $\mathrm{T}_{0}$ ) are calculated by using the ReedMerrel equations

$$
\begin{equation*}
\mathrm{L}_{0}=.276 \mathrm{l}_{0}+.724 \mathrm{l}_{1} \tag{16}
\end{equation*}
$$

and

$$
\begin{equation*}
{ }_{4} \mathrm{~L}_{1}=.034 \mathrm{I}_{0}+1.184 \mathrm{l}_{1}+2.782 \mathrm{I}_{5} \tag{17}
\end{equation*}
$$

The complete life tables are given in Table 10.

## Comments on the Life Table

The main advantage of the procedure which has been applied to calculate the life table values between ages 5 and 100 is its simplicity. A simple technique was utilized to correct the age distributions from the obvious distortions instead of the elaborate techniques of mathematical smoothing which are hard to justify in situations where systematic errors prevail. Once these obvious distortions are corrected, the proportions of the population at pivotal ages are calculated by simple methods and then multiplied by $e^{-r T}$ to give $l_{x}$, from which the rest of the life table values will follow.
Nevertheless, a comparison between the $l_{x}$ and the $\dot{\circ}_{x}$ values of the above tables and the available Egyptian life tables, which are given in Figures 7 and 8, shows clearly that the results of the method we have followed here are similar to those of the census differencing life tables of 1917-1927 and 1927-1937. We naturally have no way of telling which of the life tables is nearer to reality, but the results do establish the fact that the above simple procedure leads to a life table which is not less accurate than those given by the more elaborate methods, thus satisfying the purpose of this work.

The new life table gave reasonable results when it was compared with those of some high mortality populations. Comparison with Mexico in 1930, (8), for example, where the crude death rate was 25.6 , showed that the maximum difference between the corresponding life expectancies was about two years between ages 0 and 65 , except at ages 5 and 10 where the differences were less than four years. The life expectancies in Mexico were higher up to age 50. Similarly it was found that
the life expectancies of Egyptian males were higher than those of Indian males in 1941-1950, (4), by two years up to age 50, except at ages 0 and 1 where the Indian values were higher than the Egyptian maxima by less than one year. In the case of the females the Indian $\dot{e}_{0}$ was about half way between the corresponding Egyptian limits and $\dot{e}_{1}$ was less than one year below the corresponding Egyptian lower limit. In the rest of the table the Egyptian expectations were higher than the corresponding Indian values by between 3 and 4.5 years. It must be stated that the purpose of these comparisons is not to prove the validity of the new life table of Egypt but rather to demonstrate that this table falls, when compared with those of other countries, in a position which would be expected on the basis of the mortality situation.
As stated before, the life table values at ages 0,1 can only be taken as approximations. This is why we thought it was perhaps more plausible to estimate lower and upper limits to those values.
The life table is far from perfect. It will be noted, for example, that the values of ${ }_{5} p_{x}$ between ages 20 and 30 are high. This is a direct result of the inflation in c (30) caused by averaging the group 20-29 and the over-reported group 30-39 which was not completely corrected in the above correction procedure. In this respect we iterate that our purpose is not to calculate a perfect life table but rather to achieve the maximum possible accuracy on the basis of evidence obtained from the data without resorting to mathematical refinements.

It is also evident that the life table values given in Tables 9 and 10 are high for old ages. This inflation exists in life tables in general and results from over-estimation of old ages. It is dealt with sometimes by replacing the calculated values by extrapolated corresponding values (usually by means of a Gompertz curve). Needless to say, life table values for old ages are hardly trustworthy.

The life tables are taken to represent mortality at any time in the period 1907-1947 because of the above-discussed stabil-
ity of mortality conditions in the country throughout this period. It is possible, if one so desires, to construct a life table based on one census, 1947 for example, and the rates of growth. The resulting life table would not show any real differences from the values given here, except in so far as the difference between the average age distribution that has been used in the above calculations and the desired census age distribution can give rise to. A life table based on the 1947 age distribution was calculated and, as expected, the life expectancies did not differ from those in Table 10 by more than one year. The resulting life expectancies at age 5 , for example, were 44.1 for males and 45.9 for females as compared to 43.2 and 45.0 in Table 10.

## Birth and Death Rates

The fact that the age distribution of Egypt is approximately stable can be utilized also to calculate the birth and death rates from the $l_{x}$ column of the life table. This can be seen at once from equation (1), which gives by integration:

$$
\begin{equation*}
\mathrm{b}=\frac{\mathrm{l}_{0}}{\int_{0} \mathrm{e}^{-\mathrm{xx}} \mathrm{l}_{\mathrm{x}} \mathrm{dx}}, \tag{18}
\end{equation*}
$$

where the denominator is the area under the $\mathrm{e}^{-r \mathrm{r}} \mathrm{l}_{\mathrm{x}}$ curve, whose ordinates at the ages x given in Table 10 are obtained by multiplying the $l_{x}$ column by $e^{-7 x}$. The area under the curve is then calculated by simple numerical integration. Once we know the birth rates, the death rates will follow directly from the relation $\mathrm{r}=\mathrm{b}-\mathrm{d}$. In the present case one will naturally get maximum and minimum values of the birth rates by using the maximum and the minimum values of $\mathrm{l}_{0}, \mathrm{l}_{1}$. The results of the calculations were as follows:

Maximum Minimum
Birth rate of males (male births per 1,000

$$
\text { males })=48.1 \quad 43.1
$$

Birth rate of females (female births per 1,000

$$
\text { females })=46.1
$$

General birth rate (births per 1,000

$$
\text { population }=47.1 \quad 42.3
$$

The general birth rate is a weighted average of the male and female birth rates, the weights being the proportions of males and females in the total of the five censuses 1907-1947.
The maximum and minimum values of the death rates will be obtained by subtracting the rates of natural increase, namely $r=11.7$ for the males and $r=11.9$ for the females. The results will be:

Maximum Minimum
Death rate of males (male deaths per 1,000

$$
\text { males })=36.4
$$

Death rate of females (female deaths per $1,000$ females $)=34.2$
General death rate (deaths per 1,000 population) $=35.3 \quad 30.5$
The general death rate was calculated in the same way as the general birth rate.

It is interesting to notice that the average reported birth rate in the period 1917-1946 is equal to 42.4 , which is almost exactly equal to the above minimum value of 42.3 , while the average reported death rate is lower than the calculated minimum by 3.5 per thousand. One could say, therefore, that on the basis of the evidence secured from the reported age distributions and from the above maximum and minimum values the deficiency in birth reporting can be up to 11 per cent while the deficiency in death reporting is between 13 and 31 per cent. One must note, however, that these limits are affected to some extent by the limits we have used before for the infant mortality rate and the death rate in the age group 1-4.

## The Net Reproduction Rate and the Mean Length of a Generation

The stability of the age distribution enables us to calculate some measures of the combined effect of mortality and fertility. A first approximation to the net reproduction rate, for example, can be calculated from the approximate relation:

$$
\mathrm{b}=\mathrm{l}_{0} \frac{\mathrm{R}_{0}}{\mathrm{~L}_{0}}
$$

where $R_{0}$ is the net reproduction rate and $L_{0}$ is the area under the $l_{x}$ curve of the females and $b$ is the birth rate of the females. This relation gives $\mathrm{R}_{0}=1.4$. A thousand women passing through the child-bearing ages are therefore replacing themselves by about 1,400 baby girls. This is equivalent to a 40 per cent increase per generation.

The mean length of a generation, which is the interval during which there has been a relative increase in the female population equal to the ratio between the female births of the two generations, can also be calculated by means of the relation

$$
\mathrm{T}=\frac{\mathrm{l}}{\mathrm{r}} \log \mathrm{R}_{0}
$$

which gives a mean length of a generation equal to 28.4 years.

## Conclusion

The main motive behind this work was to look for simple methods approximating the essentials of the demographic position from data whose distortions and deficiencies do not justify the use of elaborate and detailed techniques.

The approximate stability of the age distribution furnished a useful foundation upon which the whole technique which dealt with various demographic characteristics was based. It was possible by comparing the reported age distributions to spot the obvious distortions and correct them to a reasonable extent without resorting to the more elaborate and probably unjustified methods of curve fitting. It was also possible to utilize the characteristics of a stable age distribution in constructing a life table representing mortality to a reasonable degree of accuracy by means of a simple, quick procedure. In addition, estimates of the crude birth and death rates were readily obtained.

It was even possible to estimate the mean length of a generation and a measure of replacement, namely the net reproduction rate, without using age-specific fertility.

Although the study was confined to Egypt, the technique should be applicable to populations where a stage of approxi- past experience which has already been studied and the above methods will only be useful as a simple, rapid technique for obtaining demographic measurements. The main usefulness of the method, however, will be in the underdeveloped areas that have sufficiently stable demographic conditions. There it is hoped that this method will furnish the main demographic measurements with some approximation necessitated by the nature of the available data.

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[^0]:    ${ }^{1}$ From the Office of Population Research, Princeton University.

[^1]:    ${ }^{3}$ The cohort decennial survival rate for the group aged 0-9 in 1907, for example, is defined as the ratio of the group 10-19 in 1917 to the group 0-9 in 1907.

[^2]:    ${ }^{4}$ Lotka mentions on pages $20-22$ of his book, (6), three cases where the life table $\mathrm{l}_{\mathrm{x}}$ 's were multiplied by $\mathrm{e}^{-\mathrm{rx}}$ and then used to calculate an age distribution which was then compared with the actual age distribution. The procedure followed here is the converse, namely to start with an age distribution and then multiply the proportions by $\mathrm{e}^{\mathrm{rx}}$ in order to get a life table.

[^3]:    5 In this table the 1947 proportion 90 and over was split between two groups $90-99$ and 100 and over according to the average of the ratios between the 90-99 and 100 and over groups in 1927 and 1937.

[^4]:    ${ }^{6}$ The results will in fact be identical with those obtained from a perfectly fitting curve because a third degree curve would still give the interpolation formula (9).

